John Todd Award 2022
Laudatio for Michael Lindsey

Eric Cancès

During his undergraduate studies at Stanford, Michael Lindsey distinguished himself by winning numerous accolades, most notably the Kennedy Thesis Prize for his exceptional work on optimal transport. The results he obtained, together with Yanir Rubinstein, were subsequently published in the *SIAM Journal on Mathematical Analysis* in 2017. This paper earned Michael the prestigious SIAM Student Paper Prize in 2019.

Michael then pursued his graduate studies at UC Berkeley, where his excellence continued to shine. In 2018, he was awarded the Bernard Friedman Memorial Prize in Applied Mathematics. Under the supervision of Lin Lin, he completed an outstanding PhD thesis in 2019, focusing on the quantum many-body problem – one of the most significant challenges in theoretical physics. The quantum many-body problem is central to understanding matter in all its forms, from atomic nuclei and molecules to condensed matter and Bose-Einstein condensates. Mathematically, for spin-1/2 particles on a finite lattice, the problem boils down to computing the partition function $Z = \mathrm{Tr}\left(e^{-\beta(H-\mu)}\right)$, where H is a Hermitian matrix, β is the inverse temperature, and μ is the chemical potential. The dimension of H, however, grows exponentially with the number of lattice sites L, making direct computation infeasible for systems where L exceeds 20, corresponding to matrix dimensions larger than 10^{12}. Fortunately, the algebraic structure of the quantum many-body problem provides a pathway to simplification.

The quantum many-body problem can indeed be formulated in terms of operator algebras, using the second-quantized formulation of the many-body Hamiltonian

$$\widehat{H} := \sum_{i,j=1}^{L} h_{ij}\widehat{a}_i^\dagger \widehat{a}_j + \frac{1}{2}\sum_{i,j,k,l=1}^{L} V_{ijkl}\widehat{a}_i^\dagger \widehat{a}_j^\dagger \widehat{a}_l \widehat{a}_k \qquad \text{(for two-body interactions)},$$

where $\widehat{a}_i$ and $\widehat{a}_i^\dagger$ represent the annihilation and creation operators of particles, subject to either the canonical commutation (CCR) or anticommutation (CAR) relations depending on whether the particles are bosons or fermions. The matrix h is Hermitian, and the 4th-order tensor V has certain symmetries that make the operator H self-adjoint. The first term in the Hamiltonian models the kinetic and external potential energy of the particles, while the second term describes the interactions between the particles. The goal is to compute the partition function, along with other key quantities, such as one-body Green's functions or self-energies.

A powerful approach physicists use to tackle this is many-body perturbation theory, visualized through Feynman diagrams. The simplest version involves putting a coupling parameter α in front of the interaction term and expanding the partition function in powers of α. Feynman diagrams provide an efficient way to keep track of the many terms in the expansion and can be given a physical interpretation. In addition, each diagram can be translated into a mathematical expression involving two simple objects: straight or curved lines correspond to free particle propagations described by the non-interacting Green's function G_0 (i.e. the resolvent of the matrix h), while wiggly lines represent interactions between the particles described by the interaction tensor V. At first order, there are only two diagrams: the dumbbell and the oyster, but the number of diagrams grows exponentially with the order of the expansion (Fig. 1).

However, this “bare” diagrammatic expansion is limited in practical value, as interactions are rarely small enough for perturbation theory to be effective. More useful is the bold diagrammatic expansion, describing the dynamics of “dressed” particles subjected renormalized interactions. The bold diagrammatic expansion can be formally derived from the Luttinger-Ward formalism, but this construction is not mathematically sound, and the very existence of the Luttinger-Ward functional for fermions has recently been questioned. In collaboration with Lin Lin, Michael produced a series of groundbreaking papers that provided a rigorous mathematical foundation for the Luttinger–Ward theory for Euclidean lattice fields.

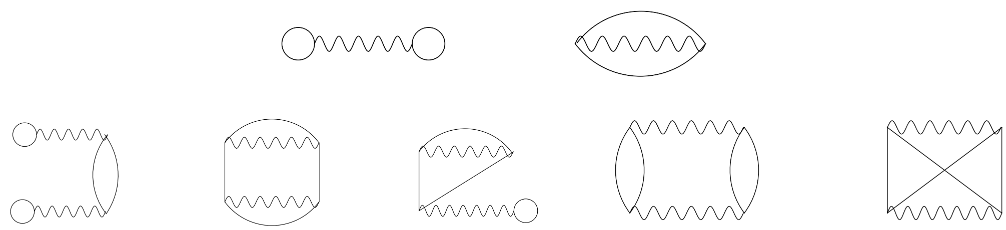

Figure 1: Bare first-order (top) and second-order connected (bottom) Feynmann diagrams.

Michael Lindsey also made major strides in quantum embedding methods. These methods provide approximations of large many-body problems by breaking them into smaller, locally solvable problems that interact through a global mean field. One notable example is the dynamical mean-field theory (DMFT), which revolutionized our understanding of the Hubbard model. Michael Lindsey proved the first rigorous mathematical results on DMFT, made improvements to numerical methods, and even introduced a novel quantum embedding method. His method, the first of its kind with a variational structure, provides a lower bound to the ground-state energy and can be solved using standard semi-definite programming (SDP) algorithms.

During his postdoctoral research at the Courant Institute, Michael Lindsey broadened his field of action to include Monte Carlo methods and tensor networks. His collaboration with top physicists notably resulted in a highly regarded paper published in *Nature Communications*. Michael Lindsey has since returned to UC Berkeley as an Assistant Professor, continuing to expand the frontiers of mathematical and numerical physics. His current research ventures into new areas such as randomized, entropically regularized semi-definite programming.

To conclude, let me quote one of Michael Lindsey’s reference letters, which perfectly encapsulates his unique talents: *In my entire career, I have not met someone who so perfectly embodies the complete package – someone for whom mathematics truly has no boundaries. Lindsey is one of the few people I know who has made contributions to both pure and applied mathematics at the very highest level.*