

Oberwolfach Prize 2025

Laudatio for Yu Deng

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Yu Deng completed his Ph.D at Princeton University in 2015 and is currently Professor of Mathematics at the University of Chicago. During the intervening decade, he has rapidly established himself as a highly original thinker whose work has already had an outsize impact on PDE theory and mathematical physics. In this laudatio, I would like to focus on two recent research programmes where Yu Deng has completely transformed the field by resolving several longstanding open problems.

The first programme concerns the construction of solutions to infinite-dimensional Hamiltonian equations with very irregular initial data. Given a polynomial P , consider for example the Hamiltonian $H_\lambda(\pi, \phi)$ defined on smooth fields $\pi, \phi: \mathbb{T}^3 \rightarrow \mathbb{R}$ by

$$H_\lambda(\pi, \phi) = \int_{\mathbb{T}^3} \left(\frac{1}{2} |\pi|^2 + \frac{1}{2} |\nabla \phi|^2 + \frac{\lambda}{4} |\phi|^4 \right) dx ,$$

which is preserved by the nonlinear wave equation $\partial_t^2 \phi = \Delta \phi - \lambda \phi^3$ if we set $\pi = \partial_t \phi$. This suggests that if we can make sense of the measure

$$\mu_\lambda(d\pi, d\phi) = \exp(-H_\lambda(\pi, \phi)) d\pi d\phi ,$$

then μ_λ should be invariant under the flow of the nonlinear wave equation. The problem with this reasoning is twofold: first, the only way to make sense of the measure μ_λ is via a “renormalisation procedure” which modifies H by an approximation-dependent diverging term $C_\varepsilon \|\phi\|^2$, so that a corresponding renormalisation procedure would have to be implemented at the level of the nonlinear wave equation. Second, the measure μ is not supported on functions, but on very irregular distributions, namely one only has $\phi \in W^{\alpha, p}$ and $\partial_t \phi \in W^{\alpha-1, p}$ for $\alpha < -\frac{1}{2}$.

A similar question in dimension two was studied by Bourgain in the mid-nineties (although he was considering the nonlinear Schrödinger equation). The two-dimensional case is much easier for two reasons. Solutions are much more regular (one only needs to take $\alpha < 0$) and the measure μ_λ is equivalent to the Gaussian measure μ_0 . Since the latter is very well understood, one has a very good handle on what the initial conditions

for the dynamic look like. This is why, despite sustained efforts by the PDE community, the three-dimensional case has long seemed out of reach.

In a technical tour de force, Deng and his collaborators Bringmann–Nahmod–Yue finally succeeded in cracking the three-dimensional case in 2024. This was the culmination of a series of articles in this general area where Deng–Nahmod–Yue refined our understanding of the two-dimensional case (with $|\phi|^4$ replaced by more general polynomials) and introduced several tools, in particular so-called “random tensors”, which have already found multiple applications.

A second research programme where Deng came out of left field to instantly impose himself as an undisputed world leader is that of deriving kinetic equations from first principles. Here, in collaborations with Hani and Hani–Ma, he solved two longstanding open problems, namely the derivation of the wave kinetic equation from the nonlinear Schrödinger equation and the derivation of the Boltzmann equation from hard sphere dynamic.

The former arises naturally when considering a rather large class of nonlinear dispersive systems at large temporal and spatial scales, and in the weakly nonlinear regime. The underlying idea is that in such a regime, one expects the relevant Fourier modes of the solution to have very rapidly varying phases so they get “smeared out” and one obtains an autonomous evolution for the amplitudes. A formal calculation allows one to “guess” the limiting equation, but placing such a formal derivation on rigorous mathematical footing has stumped mathematicians for several decades. The main reason is that the derivation effectively assumes that the phases of the Fourier modes are independently and uniformly distributed at all times. While this can simply be assumed for the initial data, it is plainly false at subsequent times.

The context of the derivation of the Boltzmann equation in dimension d is that of a collection of N hard spheres of radius ε evolving according to the usual “billiard ball” dynamic. A natural scaling is to choose these parameters such that $N \rightarrow \infty$ but $N\varepsilon^{d-1} \approx 1$. In this regime, assuming that the particles are confined in a box (or a torus) of size 1 and have initial speeds of order 1, each particle typically undergoes $O(1)$ collisions in $O(1)$ time.

If one draws the initial positions and velocities of these particles randomly and independently (modulo avoiding overlaps) from some initial distribution $f_0(x, v) dx dv$, then one expects their statistics at later times to be described by the solution f_t to the

Boltzmann equation. A mathematical justification of this derivation was obtained by Oscar Lanford in 1976 who established the convergence in question for a very short time interval (short enough so that the vast majority of particles have only been involved in at most one collision). The obstacle to extending this to longer times is conceptually similar to that arising in the wave kinetic equation: as particles are involved in multiple collisions, correlations build up and destroy the independence properties on which Lanford's result (and the formal derivation of the equation) rely. Despite sustained effort by the PDE community (which was intensified in recent years through the series of works by Bodineau, Gallagher, Saint-Raymond and their collaborators), getting these correlations under control appeared to be hopeless until the recent breakthrough by Deng–Hani–Ma.

In both cases, they showed a result that is essentially optimal, namely that the convergence of the microscopic dynamic to the solution of the limiting equation takes place as long as the latter remains smooth. The proofs in both cases rely on graphical expansions of the solution (in the spirit of Feynman diagrams). This requires not only hard analytical bounds, but also the development of a novel technique allowing to “restart” the solution at later times. The conceptual problem that has to be overcome can be best illustrated by the following fact. If one considers the state of the hard sphere system at time t and flips all velocities, then the solution after a further time t recovers the initial state exactly. If however one restarts the Boltzmann equation with initial data $\tilde{f}_t(x, v) = f_t(x, -v)$, then the solution after time t will *not* recover f_0 , thus showing that smoothness of f_t on its own cannot contain enough information to control the convergence. It was an incredible feat to be able to extract this additional information from the graphical expansions in a way that can be used effectively.

For this impressive body of work, Yu Deng has been awarded with the 2025 Oberwolfach Prize by the Oberwolfach Foundation.