Laudation

Jacob Fox completed his Ph.D at Princeton University in 2010 and is currently a Professor of Mathematics at Stanford University. Despite being in an early stage of his academic career, Fox has already earned a unique international reputation, making deep contributions in many key areas of Discrete Mathematics.

The celebrated Szemerédi Regularity Lemma, which gives a structural classification of large graphs, is a central tool in Graph Theory and in Theoretical Computer Science with applications in other areas including Additive Number Theory and Group Theory. It states that every large enough graph can be divided into subsets of about the same size so that the edges between different subsets behave almost randomly. The quantitative bounds on the number of parts one gets from the proof of the regularity lemma are enormous. One of the most important consequences of this result is the so called graph removal lemma. It states that every graph on n vertices with only $o(n^h)$ copies of a fixed graph H on h vertices can be made H-free by removing $o(n^2)$ edges. In addition to Graph Theory, this result has important applications in several areas including Additive Combinatorics, Discrete Geometry, and Theoretical Computer Science. Fox gave a new proof of the graph removal lemma that avoids Szemerédi's regularity lemma and gives a much better quantitative estimate.

The hypergraph Ramsey number $r_k(s, n)$ is the minimum N such that every red-blue coloring of the k-tuples of an N-element set contains a red set of size s or a blue set of size n, where a set is called red (blue) if all k-tuples in it are red (blue, respectively). Determining or estimating these numbers is one of the most central problems in combinatorics. The case of uniformity $k = 3$ is particularly important for our understanding of hypergraph Ramsey numbers, since there is a known construction that transforms any improvement in this case to analogous improvements for every higher uniformity k. Despite great efforts of many top researches over the last 80 years, there are still significant gaps between the known upper and lower bounds for $r_k(s, n)$. Moreover during the last 40 years there was almost no progress in obtaining better bounds for these Ramsey numbers. Fox together with Conlon and Sudakov obtained new upper and lower bound for $r_k(s, n)$ for $k \geq 3$ and fixed s, significantly improving the previous best results.

The celebrated Green-Tao theorem states that the primes contain arbitrarily long arithmetic progressions. The proof has two main parts, one of which is to establish a so called relative Szemerédi theorem. This theorem states that every relatively dense subset of a (possibly sparse) pseudorandom set of integers contains long arithmetic progressions. In his joint paper with Conlon and Zhao, Fox gives a simple proof of a strengthening of the relative Szemerédi theorem, showing that a much weaker pseudo-randomness condition is sufficient. One of the immediate advantages of this new relative Szemerédi theorem is that it simplifies the proof of the Green-Tao theorem and removes the need for the number-theoretic estimates involved in establishing the correlation condition for the almost primes. Since then, the densification technique introduced in their paper has become an important tool in the area. In particular, this technique was used recently by Tao and Ziegler to prove the existence of narrow polynomial progressions in the primes.

Jacob Fox is an extremely powerful researcher, and is already one of the leading figures in Combinatorics. For his impressive body of results, Fox is awarded the 2016 Oberwolfach prize by the Oberwolfach foundation.