OBERWOLFACH PRIZE 2019
LAUDATIO FOR OSCAR RANDAL-WILLIAMS

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Oscar Randal-Williams received his DPhil in 2009 from the University of Oxford, spent two years as a post-doc in Copenhagen, and is now Professor at the University of Cambridge. Despite his short career so far he has had a profound and far reaching impact on geometric topology through his expansive research and engagement.

Manifolds are most central objects in geometry and topology. In joint work with Soren Galatius, Randal-Williams completed a program that generalised to higher dimensional manifolds simultaneously Harer stability for mapping class groups and Madsen and Weiss's celebrated work on Mumford's conjecture. It should be stated that it was not at all clear what even the statement of an analogue in higher dimensions might be yet alone how it should be proven.

To state the most basic case, one considers even dimensional manifolds that are connected sums of g copies of products of spheres

$$W_{g,1} = D^{2n}\sharp(S^n \times S^n)\sharp \dots \sharp(S^n \times S^n)$$

Let $\mathrm{Diff}_\partial(W_{g,1})$ denote its group of diffeomorphisms that fix the boundary pointwise, and let $B\mathrm{Diff}_\partial(W_{g,1})$ be its classifying space, also thought of as a topological moduli space for $W_{g,1}$. When $n = 1$, the manifold $W_{g,1}$ is an oriented surface of genus g with one boundary component and $B\mathrm{Diff}_\partial(W_{g,1})$ has indeed the same homotopy type as Riemann's moduli space. Using parametrized surgery, a technique that they refined, Galatius and Randal-Williams prove homology stability for their moduli spaces

$$H^*(B\mathrm{Diff}_\partial(W_{g,1})) \text{ is independent of } g \qquad \text{for } * < g/2$$

and compute the rational cohomology of their limit spaces as g goes to infinity

$$H^*(B\mathrm{Diff}_\partial(W_g, 1)) \simeq \mathbb{Q}[\kappa_c \,|\, c \in \mathcal{B}] \qquad \text{for } g \to \infty$$

Here $2n \geq 6$ and $\mathcal{B}$ is the set of monomials in the Euler and Pontryagin classes

$$\{e, p_{n-1}, \dots, p_{\lceil \frac{n+1}{4} \rceil}\}$$

To put this in context, algebraic topology can claim huge successes for the study of manifolds through the hands of the early pioneers of Thom, Milnor, Smale, Novikov, and others. However, progress then became slower. A major program from the 1970s and 1980s aims to study manifolds via stable homotopy and algebraic K-theory. Here one replaces a d-dimensional manifold M by successive thickenings of itself

$$M \quad \rightsquigarrow \quad M \times I \quad \rightsquigarrow \cdots \rightsquigarrow \quad M \times I^k$$

where $I = [0, 1]$ is a unit interval. The limit of the associated moduli spaces is then computable via Waldhausen K-theory while information for M is deduced via Igusa's stability theorem, valid roughly in dimensions $* < d/3$. This puts a uniform bound on the information attainable via this methods for all d-dimensional manifolds. Furthermore, despite sophisticated tools such as the cyclotomic trace, algebraic K-theory itself remains difficult to compute.

In this new approach a different stabilisation method is used. Instead of thickening the manifold M (and hence increasing its dimension), its complexity is increased by taking repeated connected sums with a fixed manifold, such as $Q = S^n \times S^n$ in even dimensions $d = 2n$

$$M \quad \rightsquigarrow \quad M\#Q \quad \rightsquigarrow \ldots \quad \rightsquigarrow \quad M\#_g Q$$

Galatius and Randal-Williams show that the limit of the associated moduli space is computable via (tangential) cobordism theory $\mathbf{MT}SO(2n)$ which at least rationally is completely understood.

These results have already led to imaginative and surprising applications. We mention just one such application. In joint work with Ebert and Botvinnik, Randal-Williams studies the space of positive curvature metrics

$$\mathcal{R}^{>0}(M) = \{\mathfrak{g} \mid \mathrm{scal}(\mathfrak{g}) > 0\}$$

on a simply connected Spin manifold M of dimension $d \geq 6$. These spaces have initially been studied by Hitchin and more recently by Schick and collaborators. Randal-Williams and co-authors show that these spaces display a rich and highly complex topology. Surprisingly, as Randal-Williams shows in his more recent paper with Ebert, they form an object to which the powerful tools of stable homotopy theory can be applied.

In a rather different direction a third theme in Randal-Williams' research, higher algebra, provides a completely new approach to homology stability for a series of automorphism groups. In addition to a long foundational paper, jointly with Galatius and Kupers, he wrote two papers with applications to mapping class groups and general linear groups solving among other long standing problems going back 25 and 30 years. This is quite typical of Randal-Williams' work. He is not afraid to develop highly complex theory drawing on techniques from algebra, geometry or homotopy theory, and then to apply this to concrete questions often by means of hard and intricate computations.

For his impressive body of work, Oscar Randal-Williams has been awarded with the 2019 Oberwolfach Prize by the Oberwolfach Foundation.

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