Laszlo Szekelyhidi was born in Debrecen in 1977 and studied mathematics in Oxford, where he graduated in 2000, as the best student in his year. I was very happy that I could convince him to join Max Planck Institute for Mathematics in the Sciences in Leipzig, where he became my PhD student (but hardly needed any supervision) and graduated in 2003. After a postdoc position at IAS in Princeton he became a Heinz Hopf lecturer at ETH. In 2007 he was appointed as the first Bonn Junior Fellow, a position at the newly created Hausdorff Centre at the rank of Associate Professorship. In 2008 he became a member of the 'Junge Akademie' which admits ten new members outstanding researchers each year (over all fields of science and the humanities). In 2009 he his position in Bonn was made permanent.

Laszlo's work has lead to fundamental new insights into the theory of nonlinear partial differential equations and their applications in continuum mechanics. He has both constructed striking new counterexamples and developed a new structure theory which has profoundly advanced our understanding of oscillations effects in nonlinear pde. Let me focus on three highlights of his work.

Nowhere regular stationary points of polyconvex functionals in nonlinear elasticity In his thesis Laszlo obtained a result which is a milestone in the regularity theory for elliptic systems and variational integrals. He constructs a 2×2 elliptic system, which is the Euler-Lagrange equation of functional with (strictly) polyconvex energy and which admits Lipschitz solutions that are nowhere C^1 (here an energy function $f(F)$ defined on 2×2 matrices is called polyconvex if it can be written as a convex function of F and $\det F$). This is a striking result since, starting from the pioneering work of J.M. Ball, polyconvexity has been considered the natural condition in nonlinear elasticity. Also, a classical result of L.C. Evans guarantees that *minimizers* of the functional constructed by Laszlo Szekelyhidi are smooth on an open set of full measure. Thus Laszlo's work shows that general solutions of the Euler-Lagrange equations behave dramatically different from minimizers. Viewed more abstractly, Laszlo's example shows that conditions (such as strong ellipticity or even polyconvexity of the energy) which guarantee regularity for one-dimensional solutions (or for multi-d solutions with small oscillation of the gradient) do not prevent large oscillations of the gradient near every point.

V. Šverák and I had previously constructed an example of wild solutions with a quasiconvex integrand. Quasiconvexity is a natural, but much more implicitly defined and still poorly understood global ellipticity condition in the calculus of variations. It is implied by the much simpler condition of polyconvexity. The construction we used, however, cannot work for polyconvexity, due to a simple combinatorial obstruction. Thus Laszlo Szekelyhidi's result came as quite a surprise.

Compensated compactness, quasiconvexity and the Morrey conjecture In joint work with D. Faraco which just appeared in Acta Math., Laszlo Szekelehydi has obtained a breakthrough in Tartar's farreaching programme to systematically study the interaction of linear differential relations and pointwise constraints and in particular to identify necessary and sufficient conditions for compactness in this setting. Specifically they consider the following problem. Let D be the unit disc in $\mathbb{R}^2$ (any other bounded two-dimensional domain would do as well) and let K be a compact subset of the space of 2×2 matrices $\mathbb{R}^{2\times 2}$. Consider a sequence of functions

$$u^{(j)} : D \to \mathbb{R}^2 \tag{1}$$

which satisfies

$$\text{dist}(Du^{(j)}, K) \to 0 \quad \text{in } L^2(D; \mathbb{R}^{2\times 2}). \tag{2}$$

Under which conditions on K is it true that this implies that a subsequence of the gradients $Du^{(j)}$ converges strongly in L^2, i.e. satisfies

$$Du^{(j)} \to Du \quad \text{in } L^2(D; \mathbb{R}^{2\times 2})\,? \tag{3}$$

There are two known necessary conditions for compactness, an obvious one and a much more subtle one. The obvious conditions is

$$\text{rank}\,(X - Y) \neq 1, \quad \forall X, Y \in K. \tag{4}$$

This is an ellipticity condition. If it is violated, i.e., if F and $F + a \otimes b$ belong to K then compactness fails due to the presence of one-dimensional oscillations. One may, e.g., take $u^{(j)}(x) = Fx + aj^{-1}h(jx \cdot b)$, where h is a periodic Lipschitz function with slopes 0 and 1 (almost everywhere).

The more subtle condition involves a certain configuration of four matrices (often called a T4 configuration) and was independently identified by a number of authors in different contexts (Scheffer, Tartar, Aumann-Hart, Nesi-Milton, ...). The corresponding counterexample to compactness involves an iterative construction which uses oscillations on infinitely many scales. The second condition may be expressed as

$$\text{The set } K \text{ contains no four matrices } X_1, X_2, X_3, X_4 \text{ which form a T4 configuration} \tag{5}$$

The main result of Faraco and Szekelyhidi is that (4) and (5) are also sufficient to obtain compactness, i.e., (3) (for a subsequence). This is a striking result. In fact it was absolutely not clear that there exists any simple algebraic condition on K which is necessary and sufficient for compactness. Indeed, Šverák's celebrated counterexample to the Morrey conjecture (rank-one convexity implies quasiconvexity) shows that no such condition exists for maps $u : D \to \mathbb{R}^m$ for $m \geq 3$. In fact the work of Faraco and Szekelyhidi is the strongest hope so far rank-one convexity might indeed imply quasiconvexity in two dimensions, with far-reaching consequences for a number of areas.

The proof combines a careful investigation of the geometric and combinatorial properties of rank-one convex sets, which goes back to Laszlo's thesis, with a subtle use of ideas quasiconformal analysis. The results of Szekelyhidi are in my view the best results in the field since Šverak's 1992 counterexample.

Irregular solutions of the Euler equation The Euler equations of fluid mechanics have been an outstanding challenge to mathematics for more than two hundred years. The simplicitiy of their formulation hides a very rich analytical and geometric structure. One example for this was Scheffer's striking discovery in 1993 that the Euler equations have (weak) solutions which are very irregular and can even have compact support in space-time (thus strongly violating the energy inequality). Scheffer's construction is very clever, but a tour de force and has not been penetrated by many. A few years later Shnirelman gave a different construction of highly irregular solutions in a series of three papers. Recently Camillo DeLellis and Laszlo Szekelyhidi developed a completely new route to the problem (Ann. Math., 2009). They realized that by a clever reformulation of the equations the problem can be tackled by Gromov's convex integration theory (in a streamlined version due to Kirchheim, which can be used as a simple black box). Then one just has to verify certain simple facts about the linear algebra of 4×4 matrices. Their approach gives not only a much simpler proof of the old results but also opens the door to new and even more unexpected results. In particular they can show that there exist highly irregular solutions which do satisfy not only the weak form of the equation but also the distributional form of the energy inequality. Their already generated a lot of interest and among other things C. Villani gave a Bourbaki seminar on the work of DeLellis and Szekelyhidi in November 2008.

Ongoing work Currently Laszlo Szekelyhidi is working in particular on problems in hydrodynamics and the rigidity and flexibility of isometric immersions of Riemannian manifolds (and surprising connections between the two). Classical geometric results show that isometric immersions, which are sufficiently smooth are often rigid. Hilbert showed, e.g., that C^3 isometric immersions of S^2 in $\mathbb{R}^3$ are rigid motions. The striking results of Nash and Kuiper in the 50's nonetheless showed that C^1 isometric immersions can be rather 'wild' (e.g., S^2 can be mapped into an arbitrarily small ball in $\mathbb{R}^3$). Already in the 60's the conjecture appeared that the borderline between rigidity and flexibility is the Hölder space $C^{1,1/2}$. Borisov announced in the 60's that there exist $C^{1,\alpha}$ isometric immersions of n-dimensional Riemannian manifolds in $\mathbb{R}^{n+1}$ of Nash-Kuiper type for some small α, depending on n. For an analytic metric and $n = 2$ Borisov published a detailed proof in 2004. Conti, De Lellis and Szekelyhidi have recently proved the result in all dimensions and for metrics which are merely C^β, rather than analytic (arXiv:0905.0370v1). They also discuss an intriguing analogy with the Euler equations in hydrodynamics. A famous conjecture of Onsager from the 40's states that for $\alpha < 1/3$ there exist C^α solutions of the Euler equations which do not conserve energy, while solutions in C^α with $\alpha > 1/3$ are energy conserving.

In conclusion Laszlo Szekelyhidi's work shows both great depth, reflected in the resolution of longstanding conjectures, and a great breadth and the ability to make very fruitful connections between different areas of mathematics.