

REPORT ON OBERWOLFACH LEIBNIZ FELLOWSHIP

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Background. This report describes the author's stay at the Mathematisches Forschungsinstitut Oberwolfach in August-October 2010.

Research. The research I conducted during my stay at the MFO focused on two separate problems. The first was to develop a better understanding of recent work by Mukhin, Tarasov and Varchenko which relates the rational Cherednik algebra (RCA) at $t = 0$ to the Bethe algebra coming from Physics. This is part of a larger program to try and understand the connection between RCAs (and their geometric counterpart, Calogero-Moser spaces) and the affine Lie algebra $\widehat{\mathfrak{gl}}_n$ at the critical level (with its geometric counterpart, GL_n -opers on \mathbb{P}^1). During my time at MFO, I came to realize that this connection with the Bethe algebra has several important implications for the representation theory of RCAs. In particular, it implies that there is a close connection between modules for the RCA and Schubert calculus. This connection allows one, for instance, to describe the multiplicity of simple modules in certain canonically defined modules in terms of the Littlewood-Richardson coefficients coming from Schubert calculus. These new results are to appear in the preprint [B], which is in the final stages of being written up.

In addition to this, I worked on some of the combinatorial implications of a recent paper by Professor V. Ginzburg, [G], on the isospectral commuting variety (a variety closely related to the space of pairs of commuting matrices) and its relation to the Hilbert Scheme of points in the plane. This is part of a joint project with Prof. Ginzburg, which was begun during a visit to the University of Chicago earlier in the year. By exploiting these geometric results we have been able to come up with several, at present, mysterious combinatorial identities. For instance, this reasoning produces identities relating certain parabolic Kazhdan-Lusztig polynomials with expressions involving Macdonald's symmetric polynomials. My stay in Oberwolfach allowed me to work out the proofs of the main results that we have discovered. This has resulted in the preprint [BG].

Research in Pairs. During my stay at Oberwolfach, Dr. Maurizio Martino from the University of Bonn visited for a week as part of the Research in Pairs program. We discussed several problems related to the representation theory of rational Cherednik algebras and also came up with various ideas and plans of attack for these problems. Many of these ideas were found not to work but others seem quite promising. As a consequence of his visit, we currently have a joint research project exploring the representation theory and geometry of rational Cherednik algebra in positive characteristic.

Talks. I also participated in the workshop “Deformation Methods in Mathematics and Physics” and the mini-workshop “Exploiting Symmetry in Optimization”. Both of these workshops focused on topics that are only tangentially related to my own area of research. This made them extremely interesting for me since I was exposed to ideas coming from very different areas of mathematics that are none the less related to representation theory. Being at Oberwolfach, I was able to talk extensively, not only to the participants of these two workshops, but also to participants of other workshops and it was wonderful to learn about some of the problems that are of interest to mathematicians in other areas of mathematics. I believe that the Leibniz fellowship is probably unique in being able to offer young researchers this opportunity to talk with so many different mathematicians.

Thanks. I would like to take this opportunity to thank the Mathematisches Forschungsinstitut Oberwolfach for enabling me to visit Oberwolfach. My stay there was very enjoyable as well as being extremely productive and has been very beneficial for my research.

REFERENCES

- [B] **G. Bellamy**, *Rational Cherednik algebra, Schubert calculus and Bethe algebras*, *In preparation*.
- [BG] **G. Bellamy** and **V. Ginzburg**, *Some combinatorial identities related to commuting varieties and Hilbert schemes*, *arXiv:1011.5957v1*.
- [G] **V. Ginzburg**, *Isospectral commuting variety and the Harish-Chandra D -module*, *arXiv:1002.3311v1*.