Simons Visiting Professors

Activity Reports 2015

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23 January 2015

Dear Professor Huisken,

Scientific Activity Report, Simons Visiting Professorship

I am very pleased with the invitation to the Oberwolfach Workshop 'Spectral Theory and Weyl Functions', which took place 4–10 January 2015. At the workshop I gave the talk 'Analysis of the Dirichlet-to-Neumann operator on nonsmooth domains' on 5-1-2015. The talk initiated a lot of discussion, in particular with Brüning, Derkach, Gesztesy, Grubb, Hassi, Malamud, Partington and de Snoo. The talks of other participants gave me a lot of new techniques that I will explore more in the near future. The large break between the lunch and afternoon talks was excellent to discuss with other participants. Of great value are the discussions that I could have with Grubb on the Dirichlet-to-Neumann operator on domains with C^∞-boundary, in which she kindly explained the ideas scattered in various chapters in her book how this operator acts as a pseudo-differential operator.

In the period 10–25 January 2015 I was a guest at the Graz University of Technology with host Univ.-Prof. Dr J. Behrndt. We worked on abstract form methods and in particular the Dirichlet-to-Neumann operator on Lipschitz domains associated to the Laplacian with a complex potential. In a previous project with Arendt, Kennedy and Sauter the case with a real valued potential was solved. Using form methods and a technique called hidden compactness it turns out that the associated operator or graph is self-adjoint and lower bounded for real valued potentials. In a different project with Behrndt we gave an independent proof and a characterisation of the domain of this operator or graph.

The aim of the current project is to replace the real valued potential by a complex valued potential. In case the associated Dirichlet-to-Neumann graph is single valued, that is, if it is an operator, then we showed that this operator is m-sectorial. Our big surprise is that this does not extend to the multi-valued situation. We constructed a complex potential such that the associated Dirichlet-to-Neumann graph is not an m-sectorial graph. We currently investigate further properties of this graph, in particular whether this graph has a non-empty resolvent.

On 22-1-2015 I gave the seminar 'Sectorial forms and degenerate operators' in the Seminar Angewandte Analysis und Numerische Mathematik.

Acknowledgement. I am very grateful for the invitation to the Oberwolfach Workshop and for the Simons Visiting Professorship. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschunginstitut Oberwolfach. I wish to thank the TU Graz for funding the transport in Europe and the accommodation in Graz.

Yours sincerely,

Tom ter Elst

SCIENTIFIC ACTIVITY REPORT ON THE VISIT OF LIYOU ZHANG AS SIMONS VISITING PROFESSOR

LIYOU ZHANG

I have visited University of Wuppertal from 18 to 25 January, 2015, where I gave a colloquium talk from 12:30 to 14:00 pm on the afternoon of 23 January. The title of my talk is *On curvature estimates of bounded domains with the squeezing property*. During the visit at Wuppertal University for the whole week, I had some impressive discussions with the host professor N. Shcherbina on several interesting problems. Meanwhile, I attended a short courses given by Prof. A. Sukhov. I also had some nice communications with Prof. K. Diederich and Prof. G. Herbort at the host university.

From 26 to 30 January, 2015, I attended the Oberwolfach conference of *Geometric Method in Complex Analysis(1505)*, organized by Professors B. Berndtsson, J.E. Fornaess and N. Shcherbina.

What follows is a detailed description for the research collaboration with the host professor N. Shcherbina.

We talked about a new kind of biholomorphic invariant, the squeezing function, as well as some related applications in complex analysis and geometry. The concept of squeezing function was introduced in [DGZ1] in 2012. Roughly speaking, the squeezing function characterizes how a bounded domain looks like the unit ball if one observes it at one point of the domain. More precisely, let D be a bounded domain in $\mathbb{C}^n$. For any $z \in D$ and an (open) holomorphic embedding $f : D \to B^n$ with $f(z) = 0$, we define $s_D(z, f) = \sup\{r | B^n(0, r) \subset f(D)\}$ and $s_D(z) = \sup_f\{s_D(z, f)\}$, where the supremum is taken over all holomorphic embeddings $f : D \to B^n$ with $f(z) = 0$. Then s_D is called the *squeezing function* of D.

The squeezing function s_D is a biholomorphic invariant of D and takes the values in the interval $(0, 1]$. If s_D admits a positive lower bound, then D is called a *holomorphically homogeneous regular* manifold (HHR) by Liu-Sun-Yau in [LSY] or a *uniformly squeezing* domain (USq) by S.-K. Yeung in [Yeu]. The concept, HHR or USq, has been developed in order for the study of completeness and other geometric properties such as the metric equivalence of the invariant metrics. The examples of HHR or USq domains include bounded homogeneous domains, bounded strongly convex domains, the bounded domains which cover a compact Kähler manifold, and the Teichmüler spaces $\mathcal{T}_{g,n}$ of hyperbolic Riemann surfaces of genus g with n punctures (see [Yeu]). Recently, it has been proved that the bounded convex domains [KiZ] and the strictly pseudoconvex domains with C^2 boundary also admit such HHR/USq property (see [DGZ2, KiZ]).

Inspired by a recent work by Lu [Lu] and Fornaess-Wold [FoW], we aim to explore the upper and lower bounds for the holomorphic sectional curvature, the Ricci curvature and the scalar curvature on any given domain in term of the squeezing function, as well as the asymptotic boundary behaviors of these Bergman curvatures. Consequently we can obtain the estimates of the Bergman curvatures due to a very recent work on the estimate for the squeezing function, given by J.E. Fornaess and E.F. Wold in [FoW].

For further study of the squeezing function, we have chosen the following questions.

In general it is non trivial to have the explicit expression of the squeezing function on a given bounded domain, except bounded symmetric domains. For us, the only known example is the punctured ball $B^n \setminus \{0\}$, of which the squeezing function is $s_{B^n\setminus\{0\}}(z) = \|z\|$, where $\| \bullet \|$ denotes the Euclidean norm (see [DGZ1]). A natural question is: whether are the squeezing functions plurisubharmonic?

Another natural question might be: whether all bounded pseudoconvex domains in $\mathbb{C}^n(n > 1)$ admit HHR/USq property? One counterexample is the smooth pseudoconvex domain $\Omega \Subset \mathbb{C}^3$, constructed by Diederich and Fornaess [DiF], on which the Bergman metric and the Kobayashi metric are not equivalent. Consequently, Ω is not HHR or USq. However, we do not know for instance whether the bounded pseudoconvex domains of finite type are HHR/USq.

One amazing property of the squeezing function is the asymptotic behavior near the strictly pseudoconvex boundary points. Thanks to J.E. Fornaess who proposed the following interesting question: If the boundary limit of the squeezing function is 1, does this imply the domain is strictly pseudoconvex? The answer is still open now. This is also one of the questions that we have discussed during my visit to Wuppertal and that we want to work on it in the future.

Acknowledgment. I am grateful to Prof. N. Shcherbina, the host professor at University of Wuppertal, who kindly invite me to Wuppertal and Oberwolfach. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

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Scientific Activity Report

Eran Nevo

February 18, 2015

Periods: I have attended the Oberwolfach Worskhop ID 1506, on Geometric and Algebraic Combinatorics, held February 1st–7th, 2015, and right after it ended I have visited Prof. Francisco Santos at Universidad de Cantabria, Santander, on the dates February 8th–15th, 2015.

Acknowledgement: This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach, and I wish to express my gratitude to both for this great opportunity.

Scientific report

Talks

1. I have attended the many interesting talks at the Oberwolfach Worskhop ID 1506, including informal talks and an open problem session.

2. I delivered a lecture at the Oberwolfach Worskhop ID 1506, titled *Generalized Tchebyshev triangulations*, on a joint work I have done with Gábor Hetyei.

3. I gave a colloquium talk at the mathematics department in Universidad de Cantabria, titled *How many faces can a sphere have?*.

Collaborations

At Oberwolfach:

1. I had met with Karim Adiprasito and Jose Samper to discuss an ongoing project which we have started working on about a year ago, and which we plan to finish soon. It deals with a generalization of graph chordality to higher dimensional simplicial complexes, which we call *Homological Chordality*, as it depends on a certain homological property.

2. I have discussed many mathematical problems with many other participants of the workshop, exchanging knowledge and ideas. I'd like to mention that the atmosphere in the workshop was very friendly and ideal for mutual exchange of ideas and discussions. Such discussions I have held (not an exhaustive list) with: Michał Adamaszek, Bhaskar Bagchi, Basudeb Datta, Afshin Goodarzi, Michael Joswig, Gil Kalai, Kalle Karu, Martina Kubitzke, Roy Meshulam, Satoshi Murai, Isabella Novik, Vic Reiner, Ed Swartz, Uli Wagner and Volkmar Welker. Although no concrete research plans were set for the future, I expect that some of these discussions will evolve to a mutual research.

At Santander:

1. With Francisco Santos, we discussed both problems related to our recent preprint together with Stedman Wilson, titled *Many triangulated odd-spheres*, and a conjecture related to graph rigidity for multi-triangulations of polygons. The later conjecture appears in Pilaud-Santos paper *Multi-triangulations as complexes of star polygons.*

2. Two further visitors were present at the time: Jean-Philippe Labbé, who is currently a postdoc with me at the Hebrew University of Jerusalem, and Thibault Manneville, who is a PhD student at l'École Polytechnique, Paris, advices by Vincent Pilaud. Together with Paco Santos and me, the four of us presented to each other research problems of common interest, on several generalizations of the associahedron, and discussed them as a step towards collaborative research.

3. Here are some of the problems that Paco Santos and I work on, which are related to our result with S. Wilson:

A. It is known that the number of combinatorially distinct 3-spheres with n vertices is between $2^{\Omega(n^2)}$ and $2^{O(n^2 \lg n)}$. (i) Close the gap. (ii) The same bounds apply to 4-spheres, close the gap there.

B. For geodesic 3-spheres, it is known that the number of combinatorial types with n vertices is between $2^{\Omega(n^{3/2})}$ and $2^{O(n^2)}$. Close the gap. A related problem is:

C. Must 4-polytopes with n vertices have $o(n^2)$ three dimensional non-simplicial faces? (Neighborly 4-polytopes have $\Theta(n^2)$ tetrahedral faces.)

D. It is known that the number of combinatorially distinct 3-spheres with N tetrahedra is between $2^{\Omega(N)}$ and $2^{O(N \lg N)}$. Close the gap.

Note that a better lower bound in (A) would imply a superexponential lower bound in (D).

Report on Simons Visiting Professorship held at University of Wuerzburg*

Brian D O Anderson, Australian National University

Date and Location of Visit

I arrived in Wuerzburg on Friday 27th February and departed on Saturday 14th March 2015. My host was Professor Uwe Helmke, from the Department of Mathematics at the University of Wuerzburg. The visit was contiguous to a workshop entitled *Control Theory: A Mathematical Perspective on Cyber-Physical Systems* which I attended at Oberwolfach from 21 February till 27 February.

Immediate Outcomes of the Visit to Uni Wuerzburg

- Professor Helmke and I have completed two papers for submittal to the 2015 IEEE Conference on Decision and Control, to be held in Osaka, Japan in December of this year. There was a third co-author for each paper, with one of these co-authors coming from Uni Wuerzburg (Christian Lageman), with whom I had not worked before, and the other a student of mine from the Australian National University (Zhiyong Sun). An appropriate acknowledgment of support in the form requested is included in the papers.
    - Both papers dealt with problems of formations of moving vehicles, where achieving and maintaining a particular formation shape can be critical in applications. The technical difficulties revolve round having to first answer the questions of who needs to observe what, who needs to communicate with whom, and who needs to control what. The mathematical problems involve analysing coupled nonlinear differential equations in which some terms can be adjusted in an effort to achieve the desired formation behaviour.
- I presented a seminar at Uni Wuerzburg.

Further near term plans

- Some work was done involving Professor Helmke, Christian Lageman and myself, which constitutes the bulk of the work for a further paper. Dr Lageman has undertaken to prepare the first draft
- Zhiyong Sun will spend some time at Wuerzburg in July of this year; the intention is to engage in collaborative work, the content of which has been discussed.
- By way of technical background to this planned work, I note that over several years, I have worked (partially with Zhiyong Sun) on problems of 'misbehavior' of collections of vehicles required to travel in a formation. This behaviour has been observed in several different contexts, and while theoretically analysed in each of these contexts, there is no unifying theory that covers all cases, and which would additionally predict what would happen in further situations where misbehaviour is likely. The goal of Professor Helmke and myself is to provide such a general theory. The

  generation of that theory may require further time again than that planned for the visit of Zhiyong Sun.
- Professor Helmke will visit ANU in October, together with a colleague, Michael Schoenlein. We anticipate working on the 'misbehavior' problem, as well possibly as two or three other threads we have identified.

Some reflections

- By most measures I would set myself, I consider the stay at Wuerzburg to have been a success, despite its shortness. There are several reasons for this:
    - The week at Oberwolfach prior to the stay helped me at least 'warm up' for the problems, including a new one, which we tackled at Wuerzburg. For one of these problems at least, I was first exposed at Oberwolfach, and could contemplate it in the very appropriate surroundings there.
    - Professor Helmke and I know each other well, and have some experience in interacting. A relationship did not have to be built from the foundations.
    - The particular knowledge bases we each have are both complementary and overlapping. This is an ideal situation. I am a control engineer with a heavy mathematical bias, and I think Professor Helmke would describe himself as a mathematician with a heavy bias towards control theory.
- I did receive an invitation to visit Stuttgart University for one day, but decided in the light of advice concerning the associated travel burden (in time), that it would be better to spend the available time fully at Wuerzburg; two weeks is a rather short time to get anything done.

Conclusions

I would like to express my sincere gratitude to the Simons Foundation for enabling my stay at Uni Wuerzburg. Their generous support, coupled with the welcome and excellent support arrangements at Wuerzburg, allowed me to have a highly productive visit, working in close collaboration with my hosts.

Formal Acknowledgment of Support

The research stay was partially supported by the Simons Foundation and by the Mathematishes Institut Oberwolfach

Scientific Activity Report *
Simons Visiting Professor Robert R. Bruner
Homotopy Theory Workshop, March 2015

March 27, 2015

The visit was comprised of two weeks, 22 February - 8 March 2015, at the University of Oslo to work with Prof. John Rognes, followed by attendance at the Oberwolfach Homotopy Theory Workshop (1511) from 8 to 14 March 2015.

1 Progress on $THH(tmf)$

There is a filtration of the homology $H_*THH(tmf)$ with well understood filtration quotients. Prof. Rognes and I had conjectured for some time that there was an analogous filtration of $THH(tmf)$ itself which induced this filtration in homology. There was one possible sequence of obstructions to this, which would make the analysis of $THH(tmf)$ considerably more delicate to carry out, were they non-zero. During this trip we managed to eliminate this possibility. We also showed that there are unique cell complexes with the requisite homology. Together, these give us a filtration of $THH(tmf)$ by well understood tmf-modules.

We also clarified the values of several localizations of $THH(tmf)$ which we need in order to understand it integrally. Our manuscript now has a good account of this and the material described in the previous paragraph.

We also a need precise understanding of the Adams spectral sequence of the terms in the filtration and the filtration quotients. This requires more detailed information about the ordinary Adams spectral sequence converging to π_*tmf than is in the literature. We made progress on this, including independent proofs of Steenrod operations and the differentials they imply. We also give a description of tmf_* which is better suited to the methods we are using to analyze $THH_*(tmf)$ than the accounts in the literature. In the process we have produced clean accounts and proofs of a number of folklore results about tmf which will be included in the final manuscript. This part is still work in progress, and a finished account will take more time.

Our plan and expectation is that we will finish this project sometime in the summer of 2015, as we have both made it a high priority.

2 Progress on Ext software

Professor Rognes is probably the most sophisticated user of the software I have written to compute Ext for modules over the Steenrod algebra and its subalgebra A(2) (the subalgebra relevant to tmf), with the possible exception of our recently deceased colleague, Mark Mahowald.

Previous collaboration with Rognes had led to a number of improvements in the organization and reporting of the data produced by the program. This visit was similarly productive in exposing some organizational improvements to the code. These are

*This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

- improvements in the documentation, essentially a total rewrite that no longer depends upon the historical development,
- new scripts that do all the stages of the process, so that users will get usable information with one command, rather than having to execute 4 or 5 smaller steps, and
- better display options.

These have not been fully implemented, since they will take some time; the esential thing was to identify the improvements needed. Quick versions of the 2nd item and one of the display improvements was made and demonstrated during the Oberwolfach workshop.

3 Progress on Classification of $E(1)$-modules

Discussions with Prof. Benson during the Oberwolfach workshop led to a proposed plan of attack on an old claim about modules over a graded exterior algebra on two generators of unequal degree, namely that they are all sums of lightning flashes and free modules. In the subsequent week, the relevant functors needed to use the method of functorial filtrations were identified, and a proposed isomorphism between a given module and a sum of lightning flashes computed from it was written down. In the process of trying to prove that it really is an isomorphism, we found a simple counterexample. This disproves a long standing claim (Margolis, Spectra and the Steenrod Algebra, 1983, Section 18.2), resolving a question a number of us had raised. (The proof proposed by Margolis was not convincing, and no replacement that applied in complete generality had been proposed. Now it is clear why.)

The methods Prof. Benson proposed do work in the case of modules of finite type, with no boundedness assumptions. We plan to write something about this which proves the theorem in this case and exhibits the counterexample.

4 Progress on Expository Materials

Discussions with Prof. Angeltveit at the Oberwolfach workshop led to my writing an account of secondary and higher order operations in cohomology. This essentially recasts in more modern language the approach taken in 'On the Non-existence of elements of Hopf invariant one' by Frank Adams, and will probably be incorporated into a textbook in progress on the Adams spectral sequence.

Robert R. Bruner
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27 March 2015

SCIENTIFIC ACTIVITY REPORT

PHAM HUU TIEP

This is a report on my recent stay at the Mathematisches Forschungsinstitut Oberwolfach as participant in the Oberwolfach Workshop program, combined with my visit at Imperial College (London, UK) as a Simons Visiting Professor (SVP).

1. Activities

1. Visit at Imperial College, London, UK

I visited Imperial College, London, UK, for the period March 29 – April 5, 2015. My host professor was Professor Martin W. Liebeck.

During this visit, I also gave the London Algebra Colloquium on April 2, 2015 (held at City University London), entitled

"*Representations of Finite Groups: Conjectures, Reductions, and Applications*".

2. Stay at Oberwolfach

I attended the workshop "*Representations of Finite Groups*", for the period April 6 – 11, 2015. I gave a 50-min lecture entitled

"*On the McKay conjecture and Beyond*"

on April 6, 2015.

2. Scientific Report

1. Visit at Imperial College, London, UK

During this visit, I collaborated with my host professor, Professor Martin W. Liebeck, on the following three research projects.

(i) Surjective word maps and Burnside's p^aq^b-theorem

This research project is also jointly with Robert M. Guralnick (University of Southern California, Los Angeles, CA), Eamonn A. O'Brien (University of Auckland, New Zealand), and Aner Shalev (Hebrew University of Jerusalem, Israel). During my visit, we have been able to complete the project, and finalize our manuscript which will be submitted for publication soon.

In this paper, we prove surjectivity of certain word maps on finite non-abelian simple groups, which generalizes classical theorems of Burnside and Feit-Thompson. More precisely, if N is a product of two prime powers, then the word map $(x, y) \mapsto x^N y^N$ is shown to be surjective on every finite non-abelian simple group. This result generalizes the classical p^aq^b-theorem of Burnside, as well as several theorems proved recently by Liebeck-O'Brien-Shalev-Tiep, Guralnick-Malle, and Larsen-Shalev-Tiep. The second main result of the paper

Date: April 21, 2015.

states that if N is any odd integer, then the word map $(x, y, z) \mapsto x^N y^N z^N$ is surjective on every finite quasisimple group. We also prove asymptotic results on surjectivity of the word map $(x, y) \mapsto x^N y^N$ that depend on the number of prime factors of the integer N.

(ii) Characters, random walks, and coverings

This research project is also jointly with Roman Bezrukavnikov (Massachusetts Institute of Technology, Cambridge, MA) and Aner Shalev (Hebrew University of Jerusalem, Israel).

During my visit, we have been able to make a major progress on the project. This should allow us to prove a sharp bound on the complex character values for many elements of finite groups of Lie type. This result is expected to have many applications, several of which are being investigated in our project, including mixing time of random walks and diameters of Cayley graphs in finite groups of Lie type, and Fuchsian groups and Hurwitz groups.

(iii) Representations of finite exceptional groups of Lie type that are irreducible over proper subgroups

This research project is also jointly with Jan Saxl (University of Cambridge, UK).

The Aschbacher-Scott program, with the aim to classify maximal subgroups of finite simple groups of Lie type, leads to the difficult problem of classifying modular representations of finite simple groups of Lie type that are irreducible over proper subgroups. In the cross-characteristic case, that is when the characteristic of the representations is different from the defining characteristic of the group, this problem has been solved only for special linear groups (by work of Kleshchev-Tiep) and for some simple groups of small rank. Our project aims to solve this problem for all finite exceptional groups of Lie type. During my visit, we have been able to resolve several important instances of the project. We will continue our work on this long-term research project.

2. Stay at Oberwolfach

During my stay at Oberwolfach, I had fruitful discussions with various colleagues. In particular,

• I continued my collaboration with Prof. Martin Liebeck on the aforementioned research projects (ii) and (iii).

• I continued my collaboration with Prof. Gabriel Navarro (Universidad de Valencia, Spain) on our joint research projects.

• I also worked with Dr. Hung P. Tong-Viet (Universität Bielefeld) and Prof. Christine Bessenrodt (Leibniz Universität Hannover) on some joint research projects.

Acknowledgement. This research stay was partially supported by the Simons Foundation, the Mathematisches Forschungsinstitut Oberwolfach, and the NSF grant DMS-1201374. I am also grateful to Prof. M. Liebeck and Imperial College for generous hospitality and stimulating environment.

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Research Activity Report

Takuro Mochizuki

This is the report of my research activity during the stay at the École polytechnique, the Mathematisches Forschungsinstitut Oberwolfach, and the University of Mannheim as a Simons Visiting Professor.

I stayed at the École Polytechnique from April 13 through 19. During the period, I mainly discussed with Claude Sabbah (the host) and Thomas Krämer (a post-doc researcher in École polytechnique). We are mainly concerned with holonomic D-modules on abelian varieties and generalized Fourier-Mukai transforms. As their twistor enhancement, it is interesting to consider pure twistor D-modules on abelian varieties and the hyperKahler instantons obtained as their Nahm transforms. I explained Sabbah and Krämer my study on doubly periodic L^2-instantons, which are the Nahm transform of pure twistor D-modules on elliptic curves, i.e., the simplest but non-trivial case. Krämer attracted my attention to the study of the asymptotic behaviour of the hypreKahler instantons in the higher dimensional case, and he explained its significance in his study on holonomic D-modules on abelian varieties from the Tannakian viewpoint. With Sabbah, I also discussed the analyticity of the sesqui-linear pairing associated to mixed twistor D-modules.

During the stay at the École Polytechnique, on April 16, I gave a seminar talk at the IHES entitled "Quantum D-modules and mixed twistor D-modules", invited by Ahmed Abbes. At the time, I learned an interesting conjecture from Maxim Kontsevich on the description of algebraic vector bundles with an integrable connection on an affine space. I explained how the theory of mixed twistor D-modules could be applied to his different conjecture (already proved by Esnault-Sabbah-Saito-Yu and Katzarkov-Kontseich-Pantev), which I hope to be useful for further investigations. In the IHES, I also have the opportunity to attend at the lecture of Kontsevich.

I stayed at the Mathematisches Forschungsinstitut Oberwolfach, from April 19 through April 25, and I participated to the workshop "Mirror Symmetry, Hodge Theory and Differential Equations". I gave a talk entitled "Quantum D-modules and mixed twistor D-modules" on April 21.

I discussed with the other participants of the workshop, in particular, Hiroshi Iritani, Etienne Mann, Thomas Reichelt, Christian Sevenheck. Dmytro Shklyarov, and Uli Walter. It was quite useful for me to learn much about their results.

Inspired by the discussion, during the stay in Oberwolfach, I studied how my study on the mixed twistor D-module could be related to "better behaved GKZ-hypergeometric systems", and I believe that I arrived at a satisfactory understanding. I realized a description of the D-modules associated to the better behaved GKZ-hypergeometric systems in terms of the twisted de Rham complexes on normal toric varieties. On the basis of the description, I obtained the mixed twistor D-modules on some type of better behaved GKZ-hypergeometric systems. It will simplify and clarify the arguments in my preprint "Twistor property on GKZ-hypergeometric systems" in a significant way, and it will be reflected to the revision of the preprint.

I stayed in the University of Mannheim from April 25 through May 2. In Mannheim, I discussed Claus Hertling (the host) on CMC-surfaces. He provided me with his unpublished manuscript in which he gave a new language for the study of CMC-surfaces from the twistor viewpoint. It is closely related with my study on harmonic bundles from the twistor viewpoint. I expect that it might be useful for the study of the singularity of CMC-surfaces in Minkowskian 3-dimensional affine space because the twistor language has been quite useful in the study of singularity of harmonic bundles. We know various useful concepts in the study of singularity of harmonic bundles such as the parabolic weight, the residue, the limit mixed twistor structure, the Stokes structure, the reductions, etc. It would be interesting to clarify their roles in the study of the singularity of CMC-surfaces.

We also discussed with Martin Schmid (a professor in the University of Mannheim), who is an expert in the study of CMC-surfaces. He explained us the present status of the study, which was quite interesting and useful.

During the stay in Mannheim, on April 30, I gave a colloquium talk at the Heidelberg University entitled "quantum D-modules and mixed twistor D-modules", invited by Thomas Reichelt. Although the title is the same as the talks at IHES and the Mathematisches Forschungsinstitut Oberwolfach, I changed the content for the general audience.

Acknowledgement I express my sincere gratitude to Claus Hertling, Claude Sabbah and Christian Sevenheck for the invitation to the workshop "Mirror Symmetry, Hodge Theory and Differential Equations" in Oberwolfach. I heartily thank Claude Sabbah and Claus Hertling for the careful hospitality during my stay in the École polytechnique and the University of Mannheim, respectively.

This research stay was partially supported by the Simons Foundation by the Mathematisches Forschungsinstitut Oberwolfach, by the École polytechnique and by the University of Mannheim.

Scientific Activity Report of Simons Visiting Professor

Oleg Viro

May 17, 2015

As a Simons Visiting Professor, I visited the University Paris VI during the periods April 22 - 25 and May 3 - 5, before and after the MFO workshop Tropical Aspects in Geometry, Topology and Physics.

On April 24, I gave a talk at Symposium Géométrie et topologie in Jussieu. The talk was based on my recent research. The title of the talk was "Pictorial calculus for isometries"
Abstract : "In many classical homogeneous spaces any automorphism can be presented as a composition of two involutions. An automorphism is completely described by the ordered pair of involutions' fixed point sets. In low-dimensional spaces these presentations give rise to a new pictorial calculus for operating with automorphisms, which generalizes a well-known graphical presentations for vectors and translations in an affine space."

I had coversations with Johan Bjorklund, my former student, now postdoc in Jussieu. We discussed his current research project on real algebraic knots in the 3-space. It is related to the lifting problem of real rational plane curves.

I met Misha Gromov and discussed with him various topics, in particular, hyperfields and their applications.

The most exciting mathematical news for me was a current work by Grisha Mikhalkin "Quantum indices of real plane curves and refined enumerative geometry". I heard Mikhalkin's talks about this in Jussieu and Oberwolfach, and had long private conversations with Mikhalkin and Itenberg. It motivated me to revisit enumeration problems about lines and circles having special position with respect to a plane curve.

I had also a long detailed conversation with Stepan Orevkov about the status of classification problems of plane projective real algebraic curves of degree 8.

Also I had long and elaborate mathematical discussions with several other participants of the Oberwolfach workshop: Misha Polyak, Penka Georgieva, Sergey Galkin, Eugenii Shustin, Vladmir Fock, Victor Batyrev and Tobias Ekholm.

Scientific Activity Report

JULIA PEVTSOVA

This is a report on the activities which were supported by the Simons Visiting Professorship during the period from May 3 to May 16, 2015.

Dates:
1. Oberwolfach workshop 1519 "Cohomology of Finite Groups: Interactions and Applications", May 3-9, 2015.
2. University of Bielefeld, May 10-15, 2015.

Presentations:
1. *Localising subcategories for finite groups schemes*, report on the work in progress with D. J. Benson, S. B. Iyengar, and H. Krause, given at Oberwolfach, May 6, 2015.
2. *Varieties of maximal elementary subalgebras and modules of constant Jordan type*, report on a joint project with my student Jim Stark, given at the Representation Theory seminar at the University of Bielefeld, May 15, 2015.

During the Oberwolfach meeting, I was working with Dave Benson, Srikanth Iyengar, and Henning Krause on the project concerning stratification of the stable module category of a finite group scheme. This work continued while I visited Henning Krause in Bielefeld. As a tangible result of this work, we have one preprint and one paper in preparation:

(1) D. J. Benson, S. B. Iyengar, H. Krause, and J. Pevtsova, *Stratification and π-cosupport: Finite groups*, arXiv:1505.06628.
(2) D. J. Benson, S. B. Iyengar, H. Krause, and J. Pevtsova, *Stratification and π-cosupport: Finite groups schemes*, in preparation.

The main goal of this project is to prove the following theorem.

Theorem 1. *For any finite group scheme G defined over a field k of positive characteristic, there is a one-to-one correspondence*

$$\left\{ \begin{array}{c} \textit{Localizing tensor-ideal} \\ \textit{subcategories of } \mathsf{StMod}\, G \end{array} \right\} \sim \left\{ \begin{array}{c} \textit{subsets of} \\ \mathsf{Proj}\, H^*(G,k) \end{array} \right\}$$

which restricts to a one-to-one correspondence

$$\left\{ \begin{array}{c} \textit{Thick tensor-ideal} \\ \textit{subcategories of } \mathsf{stmod}\, G \end{array} \right\} \sim \left\{ \begin{array}{c} \textit{specialization closed} \\ \textit{subsets of } \mathsf{Proj}\, H^*(G,k) \end{array} \right\}$$

The representation theory of a finite group scheme G is often "wild", that is, the task of classifying indecomposable modules is hopeless. The theorem above aims to answer a coarser classification question: When can we build a module M out of module N using a fixed set of operations: extensions, direct sums, and syzygies?

In a formal language of triangulated categories, this translates into asking for a classification of localizing tensor-ideal subcategories in the stable module category $\mathsf{StMod}\, G$. For finite dimensional modules of finite groups, the second part of the theorem was established by Benson, Carlson, and Rickard [1]. They introduced techniques from algebraic topology to representation theory and, in particular, observed that to solve the classification problem even for finite dimensional modules, one has to develop new invariants for infinite dimensional ones. The first, much more general, equivalence in Theorem 1 was achieved by Benson, Iyengar, and Krause in [2] for *finite groups*. The approach used in [2] does not generalize to finite group schemes which constitute a much bigger class of interesting algebraic structures than finite groups and lack some of the essential properties that finite groups have. In our current project, we pursue a different strategy to prove Theorem 1 which relies on two recent advances in triangular geometry and representation theory of finite group schemes: the introduction of the notion of cosupport in [3] and the development of the theory of π-points in [4], [5].

A more detailed description of this project can be found in the two "long abstracts", by S. B. Iyengar and by myself, in the Oberwolfach reports issue devoted to the workshop 1519.

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

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Activity report of Peter Webb as Simons Visiting Professor to Oberwolfach

Oberwolfach Workshop attended: Cohomology of Finite Groups: Interactions and Applications 3 May – 9 May 2015 Reference Number: 1519

Host university after Oberwolfach: Université de Picardie - Jules Verne, Amiens, France (visiting Serge Bouc and Radu Stancu)

Activity at Oberwolfach 3 – 9 May 2015

The most useful part of this meeting to me was probably the conversations I had with many of the participants. I single out (in no particular order) discussions with the following people:

Ian Hambleton: discussions on equivariant topology, including a conjecture of mine on spaces with an action of a finite group satisfying conditions on the fixed points of subgroups and the orbit space.

Karin Erdmann: discussion on the existence of Auslander-Reiten quiver components with specified Euclidean tree class.

Peter Kropholler: explanation of the uniqueness of spaces EG.

Paul Balmer: discussion on Mackey functors.

Mike Hill: explanation to me of the occurrence of Mackey functors in stable equivariant homotopy theory.

The lectures were useful to me to the extent that they described new work, or work in an area less familiar to me. This did include most of the lectures given and it was very helpful to hear of these recent developments.

Activity at Amiens 11 – 15 May 2015

During the 5 days of my visit I worked with Serge Bouc and Radu Stancu on the topic of cohomological Mackey functors of finite projective dimension. We already have a paper completed on a related topic which is submitted for publication (arXiv:1503.03955). The discussions during the week were in a new direction with the goal of classifying the cohomological Mackey functors of maximal finite projective dimension in positive characteristic p (known by a theorem of Tambara to be one more than the sectional p-rank of the group). By the end of the week we had a proof of a theorem which identifies such functors with an additional condition on vanishing of Ext groups. One motivation for such work is to understand group representations better. The cohomological Mackey functors contain within themselves information about the group representations and also about how these representations behave under induction and restriction. They have potential application in the local questions of group representation theory which have dominated finite group representations in recent years. This starts a new project with these coauthors.

I also had a very useful discussion with Yann Palu on cluster-tilting objects in 2-Calabi-Yau triangulated categories.

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Scientific Activity Report

0.1 Scientific report on my stay at Oberwolfach

At Oberwolfach I worked with different people.

- With Martina Lanini in the problem of finding couter-examples to Lusztig-Dyer conjecture. This conjecture says that for an arbitrary Coxeter system W, with $x, y \in W$ and another Coxeter system W' with $x', y' \in W'$, if the posets defined by x, y (with the Bruhat order) and the one defined by x', y' are equivalent, then the Kazhdan-Lusztig polynomials are the same $P_{x,y} = P_{x',y'}$. We thought that hypercubic posets could give a counter-example, but we proved that it does not give one.
- With Ben Elias and Ben Webster we finished the annex that Webster did for my paper with Elias called ”Soergel bimodules for Unviersal Coxeter systems”. We fixed some errors (or mostly, badly explained parts) relating quantum groups with the Temperley-Lieb algebra. This paper is already accepted for publication in Transactions of the AMS.
- I worked with Geordie Williamson in some ideas that did not progress, but we had the idea during the talk of Laura Rider of finding counterexamples to the commonly believed idea that the indecomposable Soergel bimodules do not admit negative degree endomorphisms.

0.2 Scientific report on my stay at my host university

At the Max Planck institut, I worked mainly with Geordie Williamson.

The first week we were continuously trying to find an example of an indecomposable Soergel bimodule that do admit negative degree endomorphisms. Of course, this has to be in positive characteristic, because in characeteristic zero this is known not to be possible. We were trying to imitate the chose of light leaves done by Geordie in his paper ”Schubert calculus and torsion explosion”. We tried different variations of this construction, essentially because with his method (and the formula with Xua He) we can actually calculate the corresponding intersection forms.

After some days we found a candidate that seemed nice. There were negative degree morphisms in the corresponding Bott-Samelson, but we had to check that this negative degree morphisms did not come from a glueing from some indecomposable in the middle (between the Bott-Samelson and the ”bad element” we found).

Finally we were able to find the counter-example and prove that it was indeed one. The last two days we tried to generalize this to prove that you can find a degree -2 morphism in any characteristic, but we realized that we really need to understand better the construction Geordie did, in order to be able to genereliza this. We would even like to generalize it to every negative degree, not only 2. This we hope to do when he comes to Chile in September.

0.3 Period I spent at the Oberwolfach and at the host university

I got to Oberwolfach on Monday 11th May. I left to Bonn on Friday 15th May. I left Bonn on Tuesday 26th of May.

0.4 Acknowledgment

This research stay was partially supported by the Simons Foundation

and by the Mathematisches Forschungsinstitut Oberwolfach

RESEARCH ACTIVITY REPORT

VERA SERGANOVA

I participated in the workshop "Enveloping Algebras and Geometric Representation Theory" during the week 10 May - 16 May 2015. After this workshop I visited University of Nancy for the week of 16 May- 24 May 2015. This research stay was partially supported by the Simons foundation and by the Matematishes Forschungsinstitut Oberwolfach.

During my visit to Nancy I was working with my collaborator Caroline Gruson on the book about representation theory. The main idea of this book is to combine together representation theory of finite groups, Lie groups and quivers. In particular, we would like to emphasize application of quivers to different problems of representation theory. During my visit we managed to finish two chapters of the book. One of the chapters contains general facts about homological algebra and modules over rings. Another chapter is about representations of quivers. We start with definitions and examples, then explain geometry of the quiver variety and finally formulate and prove Gabriel's theorem that provides classification of quivers of finite type. We plan to finish the book in September 2015.

During my visit in Nancy I gave a colloquium talk "Tensor categories and supersymmetry".

We also started a new research project concerning W-algebras for finite-dimensional simple Lie superalgebras. We plan to work together with Caroline Gruson and Alexander Premet, three of us were at the workshop and we used this opportunity to discuss this project. Let me briefly describe what we want to do.

Finite W-algebras for simple Lie algebras were defined by Premet. In the last decade they became a very popular topic due to connections with primitive ideals, Yangians and vertex operator algebras. Let $\mathfrak{g}$ be a simple finite-dimensional Lie algebra with Lie group G. A finite W-algebra is attached to a nilpotent coadjoint orbit of G. Recall that there are finitely many such orbits. The importance of W-algebras is related to the fact that they are quantizations of the natural Poisson structures on the slices to the nilpotent orbits. On the other hand, primitive ideals of the universal enveloping algebra $U(\mathfrak{g})$ have associated varieties which are also nilpotent coadjoint orbits. Consider the family of primitive ideals of $U(\mathfrak{g})$ with given associated variety. There exists a remarkable correspondence between those ideals and simple finite-dimensional modules over the corresponding W-algebra. This correspondence was established by Premet, Gan–Ginzburg and Losev using different methods. It is the key ingredient in the proof of Joseph's Goldie rank conjecture.

The main goal of our project is to establish analogous correspondence between primitive ideals in the universal enveloping superalgebra and finite-dimensional modules over W-algebra. We believe that one can adopt Premet's method which is based on reduction to positive characteristic. Note that very little is known about primitive ideals in the supercase, therefore such a result may be useful for better understanding of these ideals.

During my stay in Oberwolfach I also discussed with Ian Musson his new idea about the proof of my conjecture in the paper "A reduction method for atypical representations of Lie superalgebras" (Adv. Math. 2003). We plan to use Ian's recent results about Shapovalov form on parabolically induced modules to prove the conjecture for the Lie superalgebra $\mathfrak{gl}(m, n)$.

I am very grateful for the opportunity to visit the institute and my colleague in France after the workshop.

SCIENTIFIC ACTIVITY REPORT

TOMOKI OHSAWA

I was a Simons Visiting Professor associated with the MFO workshop "*Mathematical Methods in Quantum Molecular Dynamics*" (Workshop ID: 1523; May 31–June 6, 2015) organized by George A. Hagedorn (Blacksburg, USA), Caroline Lasser (München), and Claude Le Bris (Marne-La-Vallee, France). Before the workshop, I stayed at Universität Tübingen (co-hosted by Stefan Teufel and Christian Lubich), and after the workshop, I stayed at Technische Universität München (hosted by Caroline Lasser). Although not officially part of my proposed activity the SVP program, I also took this opportunity to visit the University of Surrey (Guildford, United Kingdom; hosted by Cesare Tronci and Bin Cheng) as well prior to my visit to Tübingen.

Overall, all the visits were very much inspiring, and I feel very happy about coming back to the US with so many different ideas for future work going around inside my head.

Scientific Report

Stay at Universität Tübingen. I arrived at Tübingen on May 25th (Mon) and left for MFO on May 31st (Sun).

I gave a talk titled "The Siegel Upper Half Space, Symplectic Reduction, and Gaussian Wave Packet" at the Mathematical Physics seminar on May 26th (Tue); the talk is based on my recent preprint [2].

I also discussed the role of symplectic geometry in Stiepan and Teufel [5] and also in the recent work (in progress) of Stefan Teufel and his Ph.D. student Wolfgang Gaim. Specifically, we discussed the geometry behind the modified symplectic form and Hamiltonian that crop up in their works on semiclassical approximations.

Stay at Mathematisches Forschungsinstitut Oberwolfach. I arrived at Oberwolfach on May 31st (Sun) and stayed there until the morning of June 6th (Sat). I myself did not give a talk there, but talked to quite a few people as well as met potential future collaborators.

I have known George Hagedorn for about two years, and occasionally exchange ideas on semiclassical wave packets, particularly those which bear his name. About a month before the workshop, George emailed me his derivation of the generating function of the Hagedorn wave packets—which were also derived in a different way by Johannes Keller and Stephanie Troppmann (both Ph.D. students of Caroline Lasser at TU München). The email conversations with George lead me to the idea of yet another derivation of the generating function using the metaplectic group. I explained this idea to George, Johannes, and Stephanie at MFO: We have three different ways of deriving the same thing. This result also fits nicely as an auxiliary result of my work in progress [3].

I was quite intrigued by the talk of Bill Poirier (Texas Tech University, USA) on the trajectory method in quantum mechanics, and spent quite a bit of time with him discussing his results in an evening. It is my impression that the geometric aspects and Hamiltonian formulation of the theory are not well understood. I believe that the theory of the higher-order Euler–Lagrange equations on higher-order tangent bundles is helpful in understanding these aspects of the theory. I am

Date: June 19, 2015.

very happy to get to know Bill because I have been looking for chemical physicists in the nearby institutions with whom I can bounce ideas off. Given that we are in the same state (Texas, USA), I am hoping that we will be able to visit each other and also that the idea will evolve into a future collaboration.

I also found the talk of Simen Kvaal (Oslo) very inspiring; he talked about the bivariational principle in quantum mechanics. Again, I sat down with him for hours to understand the main idea behind his work, and I had the impression that there seems to be quite a bit of mathematics, particularly symplectic geometry, that needs to be understood better and fleshed out. The mathematics behind it also sounds quite similar to what is exploited in one of my papers [4]. We are both interested in keeping in touch, and talked about the possibility of visiting each other in the near future for collaboration.

Among others, I also talked to Vasile Gradinaru and Raoul Bourquin (ETH Zürich) on their numerical methods for semiclassical Schrödinger equation, Stephanie Troppmann (TU München) about her work on the non-Hermitian Gaussian wave packet dynamics, and Ben Leimkuhler (Edinburgh, UK) about the role of symplectic integrators in semiclassical settings.

Stay at Technische Universität München. I traveled from Oberwolfach to München on June 6th (Sat) and stayed there until the morning of June 13th (Sat).

I worked with Caroline Lasser and Johannes Keller at the Math Department of TU München during the week, mainly finishing up our paper [1] on a new phase space density for quantum expectations. The new phase density establishes a link between quantum expectations and classical observables in terms of probability densities with an asymptotic accuracy higher than previously known. This result not only provides a connection between quantum and classical mechanics in the semiclassical regime, but also can be implemented very easily to numerically approximate the dynamics of quantum expectations. We finished writing up the paper during my stay, and are currently proofreading the manuscript to have it ready for submission to SIAM Journal on Mathematical Analysis.

We also discussed the role of symplectic integrators in our numerical algorithm. It has been known for a long time in both the chemical physics and mathematics community that the use of symplectic integrators is essential in order to obtain reasonable numerical approximations of semiclassical observables. However, there does not seem to be a mathematical explanation of why this is the case. We started to explore this question with the working hypothesis that the preservation of the symplectic structure and the Liouville measure is the key to answering the question. While these properties are well-known features of symplectic integrators, they have never been examined carefully in the semiclassical settings such as ours. We are hoping to settle this question by continuing the collaboration.

Acknowledgments

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

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Research Report of Simons Visiting Professor James A Mingo, Queen's University (Kingston)

The purpose of my visit was to collaborate with Prof. Roland Speicher and his research group at Universität des Saarlandes. While at Saarbrücken I presented my recent work on asymptotic freeness and its consequences in a seminar and in a colloquium to the whole department.

I will begin by describing the context of my work. The study of free groups and free products of algebras has been a part of mathematics for over a hundred years. Around 35 years ago Dan Voiculescu came up with the idea of freeness in probability. While it is closely related to algebraic freeness there is a much closer parallel between independence of random variables in probability theory and free independence of random variables. The whole theory lives in this context; namely we have a unital algebra, $\mathcal{A}$, and $\varphi : \mathcal{A} \to \mathbb{C}$ a linear map. The algebra $\mathcal{A}$ is where our random variables live and φ plays the role of the expectation. When $\mathcal{A}$ is commutative this is just a reframing of classical probability theory. The real strength of free probability is that it works equally well when we tensor with matrices and become non-commutative.

In general there is little one can say about an arbitrary set of matrices; however if the matrices are in 'general position' we can say a lot. By general position we mean that they behave like elements of the free group in probability, namely after applying our expectation, which is in this case a trace. This is Voiculescu's definition of *free independence*.

In the initial work on freeness the examples were constructed 'in the laboratory' where they could be studied under controlled circumstances. A big breakthrough occurred in 1991 when Voiculescu showed that freeness occurred 'in the wild' as well—in the form of random matrices. He showed that many standard examples of random matrices ensembles long studied in multivariate statistics and nuclear physics exhibited asymptotic freeness. This meant that the freeness condition is only satisfied as the size of the matrices grows; however the approximation is so good that it has become of great practical importance.

An important problem now is find conditions when these freeness conditions apply. I have found with Prof. Speicher quite a few situations where under mild assumptions we have asymptotic freeness. In very recent work I have looked at the effect of taking a transpose or even a partial transpose. Quite surprisingly these operations free a matrix from itself; something that was quite unexpected. I presented these results in Saarbrücken and Oberwolfach and instigated some collaborations with

Prof. Speicher's research group. In addition Prof. Speicher and I made good progress towards completing a research monograph on random matrices and free probability we will publish soon with Springer-Verlag.

The visit was very productive and I am very grateful to the Simons Foundation for supporting my work.

June 26, 2015

Scientific Activity Report
Simons Visiting Professorship: July 1-4, 2015
Kiran S. Kedlaya (University of California, San Diego)

This SVP was associated to the workshop "Explicit methods in number theory" held July 6–10, 2015 at MFO. The period of the SVP was spent in the following locations, and included the following activities. (The missing date July 5 was devoted to personal travel.)

- **July 1-2**: Universität Regensburg, visiting Uwe Jannsen. I gave a number theory seminar entitled "Point counts of random varieties over finite fields: theorems and conjectures". In addition, I discussed a future collaboration with Jannsen's recent PhD student Franziska Wutz, who is planning to apply for a postdoctoral fellowship to be held at UCSD.

- **July 3-4**: Université de Rennes, visiting Xavier Caruso. I participated as a jury member in the PhD thesis defense of Caruso's student Tristan Vaccon (thesis title: "Précision p-adique").

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. Additional support was provided by the visited universities and by ProMath Arte.

At this time, there are no publications or preprints derived directly from this visit. However, as noted above, one of the purposes of this visit was to prepare for a future collaboration; any publications derived from that collaboration will include acknowledgment of this visit.

Activity Report
David P. Roberts
University of Minnesota, Morris

After participating in the Oberwolfach Workshop on Explicit Number Theory (July 6-July 11), I conducted collaborative research at ICTP in Trieste, Italy, July 12-July 25, 2015. This research stay was partially supported by the Simons Foundation and by Mathematisches Forschunginstitut Oberwolfach.

In Trieste, I collaborated with Fernando Rodriguez Villegas (ICTP) and Mark Watkins (University of Sydney). We are writing a book together with the title *Hypergeometric Motives.* In the two weeks at ICTP we focused on better understanding the underlying mathematics, writing software allowing computations on these mathematical objects, and the text of the book itself. This report gives an overview of the mathematics involved, and then in the final section indicates the progress made.

Motivic background. Let K and E be subfields of the complex numbers. Then, via André's modification of Grothendieck's original definitions, one has a category $\mathcal{M}(K,E)$ of pure motives defined over K with coefficients in E. If X is a smooth projective variety defined over K then any of its cohomology groups $H^w(X,E)$ are objects in this category. All objects are built from these via linear algebra including Tate twists. The category is identified with the category of representations on finite-dimensional E-vector spaces of a proreductive group G_K over $\mathbb{Q}$, the absolute motivic Galois group of K.

A case of particular arithmetic interest is $K = \mathbb{Q}$ and $E = \mathbb{Q}^{\rm cm}$, the union of all CM fields in $\mathbb{C}$. It is then expected (by combining several very deep conjectures) that motives $M \in \mathcal{M}(\mathbb{Q}, \mathbb{Q}^{\rm cm})$ are determined by associated L-functions $L(M,s)$ and the L-functions arising are exactly those associated to automorphic representations of the adelic groups $GL_d(\mathbb{A}_\mathbb{Q})$ having algebraic ∞-type. Thus (still assuming these various conjectures) one- and two-dimensional motives are explicitly classified via Dirichlet characters (classical) and modular forms (Wiles...) respectively.

Diagonally rational motives. The motives we study naturally lie in interesting and novel categories as follows. If K and E are say Galois number fields, then one can conjugate a given motive M in $\mathcal{M}(K,E)$ by elements $\sigma \in \mathrm{Gal}(K/\mathbb{Q})$ and $\tau \in \mathrm{Gal}(E/\mathbb{Q})$ to obtain a new motive ${}^\sigma M^\tau$. Generically a motive M has $|\mathrm{Gal}(K/\mathbb{Q})||\mathrm{Gal}(E/\mathbb{Q})|$ conjugates in this way.

Our motives lie in $\mathcal{M}(K,K)$ for a suitable abelian field K. They have the special property that M is isomorphic to ${}^\sigma M^\sigma$ for all $\sigma \in \mathrm{Gal}(K/\mathbb{Q})$. The subcategory $\mathcal{M}(K)$ of such motives has many nice features, including that some computations for general K are no more complicated than computations for the special case $\mathcal{M}(\mathbb{Q}) = \mathcal{M}(\mathbb{Q},\mathbb{Q})$. A tautological restriction-of-scalars process converts a d-dimensional irreducible motive in $M \in \mathcal{M}(K)$, different from all its conjugates, into a $[K{:}\mathbb{Q}]d$-dimensional irreducible motive RM in $\mathcal{M}(\mathbb{Q})$.

For one of the most studied classes of motives, the formalism of $M(K)$ is very enlightening. Namely Jacobi motives, as studied by e.g. Anderson and Schappacher, are rank one motives in $M(K)$. This theory only becomes truly motivic when K is allowed to be totally imaginary. For example for $K = \mathbb{Q}$ it is essentially the theory of quadratic Dirichlet characters.

Hypergeometric and related motives. The main focus of our book is the explicit study of a certain class of motives in $\mathcal{M}(K)$, namely hypergeometric motives (HGMs). We also explicitly study two other classes of closely related motives, also in $\mathcal{M}(K)$. For all these motives $M \in \mathcal{M}(K)$, we aim to compute the invariants associated to each place of K which are relevant to the L-functions $L(RM,s)$. In favorable cases we make connection with automorphic forms, but more typically we view that M has been adequately studied when we succeed in numerically verifying that $L(RM,s)$ satisfies the expected analytic continuation and functional equation.

1. HGMs. Let $\alpha_1, \ldots, \alpha_d, \beta_1, \ldots, \beta_d$ be elements in $\mathbb{Q}/\mathbb{Z}$ with no α_j equal to β_j. Write $a_j = \exp(2\pi i \alpha_j)$ and $b_k = \exp(2\pi i \beta_k)$. Form the rational function $\prod(x-a_j)/\prod(x-b_k)$. Let K be the field of coefficients of this rational function. Then for $t \in \mathbb{Q} - \{0,1\}$ one has a motive

$$H(\alpha, \beta, t) := H(\alpha_1, \ldots, \alpha_d; \beta_1, \ldots, \beta_d, t) \in \mathcal{M}(K) \tag{1}$$

The term hypergeometric is used because the entries in the period matrices of these d-dimensional motives are given by hypergeometric functions.

A quick idea of the nature of hypergeometric motives is given by their associated Hodge vectors $(h^{w,0}, \ldots, h^{0,w})$, which can have no interior zeros. When $K \subset \mathbb{R}$, the vector needs to satisfy Hodge symmetry $h^{p,q} = h^{q,p}$. The broad scope of the class of HGMs is illustrated by the fact that for the important case $K = \mathbb{Q}$, all such Hodge vectors in dimension $d \leq 20$ arise.

2. Special HGMs. Taking $t = 1$ in (1) gives motives $H(\alpha, \beta, 1)$ of dimension $d-1$ or $d-2$. These motives are particularly interesting in that their conductors tend to be very small, and there can be one or two zeros in the middle of their Hodge vectors, as in $(1, 1, 0, 0, 1, 1)$.

3. Moment HGMs. Let j be the inclusion of the punctured t-line $\mathbf{P}^1 - \{0, 1, \infty\}$ into the full t-line $\mathbf{P}^1$. Let M be a motivic local system on $\mathbf{P}^1 - \{0, 1, \infty\}$. Then, for a linear algebra operation S such as symmetric power, one has a natural moment motive $H^1(\mathbf{P}^1, j_* SM)$. Applying this construction to $M = M(\alpha, \beta)$ gives particularly interesting motives. Again conductors can be very small, and sometimes the Hodge vector is very spread out, like $(1, 0, 0, 1, 0, 0, 1, 0, 0, 1)$.

Issues of rationality. For HGMs, special HGMs, and moment HGMs, there is such a wealth of examples with reflex field $K = \mathbb{Q}$ that it may seem almost like seeking trouble to allow general reflex fields. Indeed most of the examples that we are explicitly pursuing have reflex field $\mathbb{Q}$. However, there are two reasons that we are systematically allowing general K.

First, while the degeneracy of the motivic family $H(\alpha, \beta)$ at $t = 1$ is mild, it is severe at $t = \infty$ and $t = 0$. When $H(\alpha, \beta)$ is defined over $\mathbb{Q}$, the description of the degeneration at $t = \infty$ involves Jacobi motives $J_j \in \mathcal{M}(\mathbb{Q}(a_j))$. Similarly, at $t = 0$, one needs to involves Jacobi motives $J_k \in \mathcal{M}(\mathbb{Q}(b_k))$. Thus, even to properly understand examples in $\mathcal{M}(\mathbb{Q})$, one needs to enter into the full formalism.

Second, the general case of $\mathcal{M}(K)$ really is the natural setting and includes new phenomena. One of the new phenomena is that motivic Galois groups regularly involve not only the usual symplectic and orthogonal groups, but now also unitary groups.

The two weeks at ICTP. We progressed in several mathematical ways towards our book. Here are three, with some indications as to how our project connects with other work.

- Different varieties can serve as sources of our hypergeometric motives. We studied this issue further, with a recent preprint of Beukers, Cohen, and Mellit playing an important role.
- We incorporated Jacobi motives much more thoroughly into our project, allowing us to systematically work with general K. Our new `Magma` package for Jacobi motives leaves room to theoretically treat bad reduction in a way that will be transferable to HGMs. Helpful work with respect to bad reduction includes a new perspective by Tsushima on classical results, with factors in the key formula being made explicit in two papers by Sharifi.
- We began code for computing L-functions of moment HGMs, explicitly working out some first cases. Interestingly, for moment HGMs one can start from hypergeometric systems which are nonmotivic, in that the number of α_j's is different from the number of β_k's. This makes more parts of Katz's very extensive work relevant to us; a recent paper of Yun provides a framework for some of these nonmotivic-to-motivic examples.

Considerable time was also dedicated to merging our various drafts of sections, so that the book as a whole is starting to take shape.

Scientific Activity Report

Simons Visiting Professorship Program

Marek Biskup (UCLA)

Oberwolfach workshop: Interplay of Analysis and Probability in Applied Mathematics
Dates at workshop: July 26-31, 2015

Host Institution: Weierstrass-Institut für Angewandte Analysis und Stochastik, Berlin
Period spent at host institution: July 19-20, August 1-8, 2015

Report on activities

The majority of the time I spent at the Weierstrass-Institut (WIAS) has been devoted to extensive discussions with my host, Wolfgang König, and one of the WIAS members, Renato S. dos Santos. The project we tackled together concerns a precise description of the solution to the so-called parabolic Anderson problem with random i.i.d. potentials whose upper tails lie in the universality class of doubly-exponential distribution. The parabolic Anderson problem is ubiquitous in many parts of science as a model that combines natural diffusive behavior of constituents with spontaneous growth/decay processes governed by spatially inhomogeneous (random) field.

The class of doubly-exponential upper tails has been known for a while to be the principal new challenge in this subject area. The reason is that the leading eigenvectors corresponding to the Anderson Hamiltonian — the operator that drives the evolution — spread over the whole lattice. Earlier work dealt only with heavier upper tails, where the eigenvectors are supported more or less at a single lattice site. It appears that, thanks also to some earlier work that I have done on the spectrum of such operators jointly with W. König, we have finally been able to overcome all technical challenges and thus put the parabolic problem with doubly-exponential tails under control as well. A paper is now in preparation that will sum up our findings. The acknowledgment of support from the SVP program, as well as the MFO, will be more than in order.

Other activites during my visit included discussions with other members of the WIAS as well as the larger mathematical community in Berlin. In particular, a prospective collaboration has been started jointly with Martin Slowik (TU Berlin) and Franziska Flegel (graduate student at WIAS) concerning homogenization of the spectra of random conductance models with conductances having power-law (lower) tails at zero. It appears that we may have a way to show that the power 1/4 (for the asymptotic behavior of the distribution function) is critical for appearance of localized principal eigenfunction in the spectrum; for lighter tails the classical homogenization picture takes over. Plans have been hatched for F. Flegel's research visit to UCLA in the Fall 2015.

Further productive interactions took place at the MFO workshop itself. I delivered a lecture on "Two recent Wulff-shape theorems" where I described some recent work of mine and collaborators' dealing with limit shapes in particular contexts of probability and statistical physics. In the discussions after my talk, I found it striking that analysts working on some homogenization problems had tackled similar questions before albeit using very different means, and naturally, with different formulation of the outcomes. I felt that the workshop served well the intent expressed in its title.

Note on timeline

Due to previous engagements, the itinerary of my trip was somewhat complicated so let me explain this in some detail: I took a 4 day break (July 21-24) during my visit to Berlin to participate in the 60-th Birthday Conference of Bálint Tóth in Budapest. I spent Saturday, July 25, sightseeing in Berlin and Sunday, July 26, traveling by train to Oberwolfach. I returned to Berlin right after the meeting (July 31). I left Berlin on August 8.

The acknowledgement

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. I wish to express my thanks to both institutions for letting me have this opportunity.

Sincerely yours,

Marek Biskup

Scientific Activity Report

Gordon Slade, Simons Visiting Professor

This is a report on my scientific activities during visits to Université de Montpellier (August 22-30, 2015) and Mathematisches Forschungsinstitut Oberwolfach (August 30 - September 4, 2015).

1. Visit to Université de Montpellier

I arrived in Montpellier on August 22, 2015 and departed on August 30, 2015.

My visit was at the invitation of Professor Damien Calaque. Prof. Calaque contacted me in February 2015 concerning his plan to organise (with Dominique Manchon of the Université Blaise Pascal) a conference on renormalisation, with an invitation to give a series of lectures at the conference. In view of my plans to visit Oberwolfach August 30 to September 4, the conference was scheduled one week prior, and I was very pleased to participate.

The conference was entitled "Renormalization in statistical physics and lattice field theories" and was held at the Institut Montpelliérain Alexander Grothendieck, August 24–28, 2015. It was supported by the GDR "Renormalization", by the Institut Universitaire de France, and by the Simons Foundation via your support of my visit. I gave a series of four one-hour lectures on the subject "Renormalisation group and 4-dimensional critical phenomema," based on recent joint work with Roland Bauerschmidt and David Brydges. Other lecture series were given by Vincent Rivasseau and Uwe Täuber, and several other participants gave one-hour lectures.

The workshop was small by design, and there were both mathematicians and physicists participating. There was a lot of coherence in the three lecture series, and there was a lot of discussion and interchange of perspectives. I found the communication between mathematicians and physicists to be particularly effective, and for me the conference was a valuable learning experience, as well as an opportunity to present my own work.

2. Visit to Mathematisches Forschungsinstitut Oberwolfach

I arrived at MFO on August 30, 2015 and departed September 4, 2015, to return immediately to Vancouver where my teaching term will begin.

I participated in the workshop "Scaling Limits in Models of Statistical Mechanics" organised by Dmitry Ioffe, Gady Kozma, and Fabio Toninelli. Workshops on this topic have been held at MFO every few years, and I have participated also in the past, most recently in 2012. These workshops bring together a community of scientists with common interests. This community is a strong network of researchers who have come to know each other well over the years, partly due to repeatedly meeting in the special atmosphere at MFO which is highly conducive to scientific discussion and collaboration. It was also encouraging to meet the excellent young people who are entering the field and are already contributing at a high level.

I gave a lecture on September 2nd, entitled "Critical behaviour of spin systems and weakly self-avoiding walk in dimension 4," based on joint work with Roland Bauerschmidt, David Brydges, and Alexandre Tomberg. An abstract was submitted to Oberwolfach Reports.

The lectures by other participants exposed me to new ideas and brought me up to date on recent developments. I was able to continue my research collaboration with Roland Bauerschmidt (Harvard University), and we made progress on two research projects we are working on. Both involve applications of our recently developed renormalisation group method. I also had useful scientific discussions with Michael Aizenman, Remco van der Hofstad, and many others. In addition, with Omer Angel we were able to recruit two of the participants as the two main speakers for a proposed PIMS Summer School in Probability to be held at the University of British Columbia in June 2017.

So in addition to being comfortable and pleasant, it was also a very productive visit to Oberwolfach.

3. Acknowledgement

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. I am grateful to the Simons Foundation for its support, and to Damien Calaque for the invitation and kind hospitality in Montpellier. My travel was supported in part by NSERC of Canada.

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Oberwolfach, September 4, 2015

Report of the SVP- MFO

Neela Nataraj

1. Introduction

I attended the Oberwolfach Workshop *MS 1540: Computational Engineering* from *27th September to 3rd October 2015.* This workshop was organised by Professors Susanne Brenner, Carsten Carstensen, Lesek Demkowicz and Peter Wriggers.

2. Visit to Oberwolfach

I arrived in Oberwolfach on 27th September 2015. The workshop MS1540 had participants who were Applied Mathematicians, Numerical Analysts and Engineers working on Computational PDE.

During my visit, I attended thirty eight talks and mini-talks organised in the workshop. I also delivered a talk entitled *Finite Element Methods for the Von Kármán Equations* on Wednesday, the 30th of September 2015 from 9.30 A.M.- 10 A.M. The talk focussed on a joint work with a doctoral student Gouranga Mallik on conforming and nonconforming finite element methods for approximation of solutions of fourth order nonlinear plate problems. A report on this has been sent to the Organisers.

I had the opportunity to interact with some participants of the workshop. In particular, I had work dicussions with Professors Susanne Brenner, Li-yeng Sung (Louisiana University), Carsten Carstensen (Humboldt Universität zu Berlin), Stefan Sauter (Universität Zürich) and Thirupathi Gudi (IISc Bangalore).

An ongoing joint work entitled *A nonconforming finite element approximation for optimal control of an obstacle problem* with Dr. Thirupathi Gudi and Ms. Asha K. Dond (a graduate student from IIT Bombay working under my supervision) was finalised during the visit to MFO and communicated to a journal for a possible publication. The SVP visit has been acknowledged in the research paper. The discussion with Prof. Sauter has initiated possibilities of joint research work in the future.

3. Visit to Humboldt Universität zu Berlin

After the visit to MFO, I arrived in Berlin on the evening of 3rd October, 2015 and stayed here for a week. During my visit, I visited the Humboldt Universität zu Berlin.
The main academic activities can be summarised as follows:

- Presented a one hour talk entitled *Finite Element Methods for Distributed Optimal Control Problems Governed By Plate Equations* on the 7th of October 2015 from 10 AM-11 AM.

- Had discussions with Prof. Carstensen on an ongoing joint research work on *A posteriori error control of conforming, nonconforming and discontinuous Galerkin finite element approximation to nonsingular solutions of the Von Kármán Equations.* The support of SVP will be acknowledged when the paper is communicated.
- Had academic interactions with Prof. M. Hintermüller who works on Optimization. The possibility of collaborations in the future was discussed. One possiblity we plan to explore is to write a joint DAAD project.
- Had a discussion with Karoline Koehler, who is a graduate student of Prof. Carstensen and works on conforming, nonconforming and mixed formulations for obstacle and Signoroni problems. Karoline had visited India before and she was also invited to MFO: MS 1540 by the organisers. During my visit to Berlin, I discussed more details of her thesis and have been asked to become part of the thesis examination by her Ph.D. advisor.

4. Visit to Heidelberg

I visited the Heidelberg University between 10th October 2015 and 17th October 2015. My host in the University was Prof. Guido Kanschat from IWR, Heidelberg University. I had several interactions with his research group and also gave a talk here on 15th October 2015 in the topic *Conforming and nonconforming finite element methods for plate problems.*

I also had several discussions with Prof. Kanschat on development of error estimates for nonconforming and mixed formulations for second order linear elliptic problems defined in polygonal domains using the rectangular Rannacher-Turek finite elements. We discussed about a plan of Prof. Kanschat's visit to India in Spring 2016, where he would conduct a workshop in IIT Kanpur focussing on finite element methods using deal.II, for the graduate and engineering students.

5. Visit to Kaiserslutern

Though not a part of SVP programme, I will be visiting the University of Kaisersluatern in Germany till 23rd October 2015. My academic host is Prof. Axel Klar. I plan to interact with the appied mathematicians and numerical analysts of the department. I plan to teach a four hour compact course on *Finite element methods for disributed optimal optimal control problems governed by partial differential equations.* This course will be attended by some graduate students of the department.

6. Acknowledgments

The research stay in MFO, Berlin and Heidelberg was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Simons Visiting Professor Activity Report

Erin Wolf Chambers*

The SVP supported my visit to TU Eindhoven for a week following my attendance at the MFO Workshop on Computational Geometric and Algebraic Topology; this stay lasted from 10 October to 24 October, 2015.

First week

The initial week of this visit was spent attending the actual Oberwolfach workshop. Activities of course included attending talks and collaborating with a variety of researchers. I also presented a survey talk, "Topological Measures of Similarity for Curves on Surfaces" on Wednesday of the workshop.

As far as collaboration, of particular note are two new projects begun at the workshop, both resulting from discussions following my talk:

Computing homotopy height. I began discussing the problem of calculating the minimum height homotopy in a discrete setting with Dr. Arnaud de Mesmey, another workshop attendee from France. He recently discovered a PhD thesis by Gregory Chambers which focused on the same problem in the continuous Riemannian setting. While there are similarities, the techniques in this thesis do not immediately carry over to the discrete setting. However, the key notion in the continuous setting is to change the homotopy into an isotopy with height almost the same as the initial one, after a natural assumption on generality is applied. It is likely that this approach will generalize to the discrete setting, which would imply that such homotopies are monotone and hence contained in the complexity class NP. We plan to investigate these techniques in the coming months over Skype, in the hopes of translating their key notion to our setting.

Related to this problem, I am examining the possibility of using harmonic forms to make an LP formulation of the homotopy height or homotopic Fréchet problem, which again were two of the variants I discussed in my survey. This collaboration is with Dr. Primoz Skraba and Dr. Tasos Sidripolous, two other attendees to the workshop. We plan to email and Skype to discuss this LP formulation as well as the possibility of it leading to an efficient algorithm for these problems in limited settings such as the boundary of a topological cylinder, in either the continuous or discrete setting.

Applying homology area. Another portion of my talk was to present recent work, joint with Dr. Mikael Vejdemo-Johansson who was also at the workshop, giving an efficient algorithm to

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calculate the "homology area", or minimum area chain co-bounding two input cycles on a surface. Following the talk, we were approached by Dr. Florian Pokorny, also attending the workshop, to ask if we could extend the method to work for relatively homology classes in the plane. In his application, he has GIS trajectory data which is being clustered by persistent homology, and he hopes to extract a central medial trajectory as a representative for each class. As homology area is extremely efficient in this case, pairwise similarity comparisons in order to extract a representative is a natrual application of our result. We have exchanged necessary code, and plan to run simulations and continue to collaboration with an end goal of submitting a paper analyzing this new method by the end of the year.

Second week

The second week was my research visit, hosted by Dr. Bettina Speckmann at TU Eindhoven. While visiting, I gave a talk to her research group on Tuesday entitled "Computing max flow in 1-crossing-minor-free graphs", which focused on efficient maximum flow algorithms in a natural minor free graph family.

The collaboration with her lab in Eindhoven is based on examining both topological measures for similarity of curves and applications for these measures in the field of GIS. Our main focus for this visit was trying to develop an algorithm for the isotopic Fréchet distance, a measure which has no known algorithm to compute it. We focused on the setting of curves in Euclidean space as a starting point, and discovered interesting connections to graph drawing and morphing of planar graphs. We are continuing this collaboration to finish developing this algorithm, and hope to publish it in the next year.

Finally, I also spoke with Dr. Maarten Löffler of Utrecht University, continuing a collaboration we began last year. We are working on using the homotopy area result in my prior work to find a representative medial trajectory from a set of input trajectories. While we have proven that the most general forms of this problem are NP-Hard, we have also been able to develop algorithms for some stricter settings, such as when the underlying graph of the trajectories is a DAG. We are finishing the writing for this work, and will be submitting to the Symposium on Computational Geometry in December.

Scientific Activity Report – Simons Visiting Professorship

Jonathan Spreer

November 4, 2015

Dates

Institution	Dates	Presentation(s)
MFO	11/10/2015 – 17/10/2015	*Collapsibility and 3-Sphere recognition* [4]
		The GAP-package simpcomp [2]
IST Austria	19/10/2015 – 24/10/2015	*Algorithms and Complexity for Turaev-Viro invariants* [1]

Oberwolfach workshop on Computational Geometric and Algebraic Topology

The main idea of the Oberwolfach workshop on *Computational Geometric and Algebraic Topology* was to bring together three closely related, yet largely disjoint research communities. As a result of this objective I was able to engage in numerous discussions obtaining valuable insights and inspirations on how to tackle important problems in my area presented from a different point of view. Most notably:

- With *Arnaud de Mesmay*: initial discussions about computing Khovanov homology of knots as a possible approach to find a fixed parameter tractable algorithm for unknot recognition. This idea has strong links to my presentation about collapsibility at the workshop, see [4].

- With *Eric Sedgwick*: discussions about surfaces embedded in 3-manifold triangulations of so-called "helical" type. These surfaces occur naturally *(i)* when computing Turaev-Viro 3-manifold invariants, and *(ii)* as the splitting surface in a Heegaard decomposition of a 3-manifold triangulation. This further motivates my plan to investigate such surfaces coming from Turaev-Viro invariants in order to obtain structural results about 3-manifolds. Eg. lower bounds on the minimum size of a triangulation of a particular topological type. This is closely related to my talk given at IST Austria.

- With *Benjamin Burton*: Discussions about using persistent homology (the core research technique of one of the three research communities brought together at the MFO workshop) to analyse the space of all 3-sphere triangulations, the so-called Pachner graph. This project, as well, has close links to my presentation at MFO [4].

In addition, I continued my ongoing collaboration with Hyam Rubinstein and Stephan Tillmann on combinatorial techniques to detect and analyse hyperbolic structures on 3-manifolds.

Visit at the Institute of Science and Technology (IST) Austria

During my stay at IST Austria I collaborated with Uli Wagner and his PhD student Kristóf Huszvar on describing topological parameters on a 3-manifold M – such as the *linear width* as defined by Scharlemann and Thompson in [5] – and their connection to graph theoretical parameters on the dual graph of triangulations on M.

Background: This research is motivated by the search for efficient algorithms to solve high-profile problems in computational topology. Many topological problems of 3-manifolds can efficiently be solved for input triangulations having a dual graph of bounded treewidth. Thus, if every triangulation could (efficiently) be transformed into a triangulation with dual graph of bounded treewidth, all these problems would be polynomial time solvable.

This is not expected to be true. One of the many reasons for this belief is the fact that this would have enormous complexity theoretical consequences. However convincing and rigorous arguments disqualifying this possibility are yet to be found. One such argument would be the explicit description of an infinite family of topological 3-manifolds $(M_i)_{i \in \mathbb{N}}$ such that the minimum treewidth $tw_i(\Gamma(T))$ of the dual graph $\Gamma(T)$ ranging over all triangulations T of M_i tends to infinity as i grows larger. For this the graph theoretical parameter defined on a single triangulation must be transformed to a topological property of the underlying manifold.

Preliminary results suggest that we can *(i)* relate the topological property of linear width of a 3-manifold to the combinatorial property of pathwidth of the dual graph of its triangulations, and that *(ii)* 3-manifolds with arbitrarily high linear width indeed exist. Similar connections and results are expected to hold for treewidth, but more work needs to be done to obtain a proof.

This work in progress is closely related to Problem 8 about knot diagrams presented at the problem session held one week earlier at the MFO workshop on *Computational Geometric and Algebraic Topology.*

Acknowledgement

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. Furthermore, the author wants to thank IST Austria for their hospitality and their support.

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Activity Report

23 October 2015

My research stay at the University of Klagenfurt, as a Simons Visiting Professor, hosted by Professor Angelika Wiegele, 4–18 October 2015, and my period at MFO, 18–23 October 2015, were extremely successful.

Professor Wiegele and I initiated some new research on "Gomory mixed-integer" (GMI) cuts for mixed-integer linear optimization. Developing a kind of generalized "basic mixed-integer" (BMI) cut via a disjunctive argument, we found a way to avoid the dual simplex method, completely working with the primal simplex algorithm, arriving at a finitely-convergent column-generation algorithm for mixed-integer linear optimization, based on a variation of GMI cuts. We already finalized our work and distributed it on the arXiv, as well as having submitted it to be considered for journal publication.

Secondly, Professor Wiegele, Professor Rendl and I have initiated some new work on a strengthened reformulation/convexification scheme for the well-known max-cut problem. Based on semidefinite optimization, a scheme for turning a max-cut problem into a *convex* quadratic ± 1-minimization problem was already known. In such a manner, a solver with the capability of handling generic binary convex quadratic minimization, such as Cplex, can be employed. We devised a way to strengthen the convexification by including dual information from triangle inequalities. In the end, we get a tighter convexification, though it is a relaxation rather than a precise reformulation at the ± 1-valued points. Still, we can devise a branch-and-bound scheme based on our convexification, however we cannot use a solver such as Cplex as a black-box, because it will miscalculate cut values of ± 1-valued points. There is a lot of room for refining our approach, for example (i) using other inequalities besides triangle inequalities, (ii) taking advantage of QP "hot starts", and (iii) variable fixing via dual variables. We have plans to test our ideas in a specialized C++ code that we are developing.

Finally, Professors Rendl, Wiegele and I are reviving an idea that Professor Rendl and I had a few years ago for employing disjunctive cuts for nonconvex QP to attack better the max-cut problem. We developed a novel cut-generating semidefinite program. We plan to test this idea in Matlab. This is quite a tricky matter as there are subtle issue regarding normalization of for the cut-generating semidefinite program that need to be addressed.

Additionally, (i) I gave lectures at the University of Klagenfurt and at MFO, on *comparing polyhedral relaxations via volume*, (ii) I advised a Ph.D. student at Klagenfurt on a possible idea for making some streamlined convex-hull like calculations in the context of vertex packing, and (iii) I interacted with many Ph.D. students and faculty at Klagenfurt, and of course the many participants at the MFO workshop, learning and exchanging about research.

This research stay was partially supported by the Simons Foundation and by Mathematisches Forschungsinstitut Oberwolfach.

Simons Visiting Professorship Activity Report

Sven Leyffer, Argonne National Laboratory

Scientific Report. The Simons Visiting Professorship enabled me to deepen my collaboration with Professor Sebastian Sager at the Universität Magdeburg. In particular, we are establishing a new research thrust in optimization which is a direct result of the Oberwolfach workshop. We are interested in optimization problems constrained by partial differential equations (PDEs) with integer decision variables. This new class of mathematical problems, called mixed-integer PDE-constrained optimization (MIPDECO) [4], must overcome the combinatorial challenge of integer decision variables combined with the numerical and computational complexity of PDE-constrained optimization.

Examples of MIPDECOs include the remediation of contaminated sites and the maximization of oil recovery, which involve flow through porous media and the optimization of wellbore locations and optimal flow rates [5], and operational schedule [1]. Related applications also arise in the optimal schedule of shale-gas recovery [7]. Next-generation solar cells face complicated geometric and discrete design decisions to achieve perfect electromagnetic performance [6]. In disaster-recovery scenarios, such as oil spills [8], wildfires [2], and hurricanes [3], resources need to be scheduled to mitigate the disaster while adjusting to the underlying dynamics for accurate forecasts. Other science and engineering examples include wind farm design [10], and the design, control, and operation of gas networks [9].

All these applications have in common that they combine discrete decision variables with complex multi-physics applications. Until recently, these grand-challenge problems have been regarded as hopelessly intractable. Together with Sebastian, we are starting to dispel this wide-held belief, motivated in part by emerging exascale computing resources. Formally, we state a mixed-integer PDE-constrained optimization problem as

$$
\begin{aligned}
\underset{u,w}{\text{minimize}} \quad & \mathcal{F}(u,w) && \text{(1a)}\\
\text{subject to} \quad & \mathcal{C}(u,w) = 0, && \text{(1b)}\\
& \mathcal{G}(u,w) \le 0, && \text{(1c)}\\
& u \in \mathcal{D}, \quad \text{and} \quad w \in \mathbb{Z}^p \quad \text{(integers)}, && \text{(1d)}
\end{aligned}
$$

which is defined over a domain Ω. We use x, y, z to indicate spatial coordinates of the domain Ω, and t to denote time. The objective function of (1) is $\mathcal{F}$, $\mathcal{C}$ are the equality constraints, and $\mathcal{G}$ are inequality constraints. The equality constraints include the PDEs as well as boundary and initial conditions. We denote the continuous decision variables of the problem by $u(t, x, y, z)$, which includes the PDE states, controls, and design parameters. We denote the integer variables by $w(t, x, y, z)$, which may include design parameters that are independent of (t, x, y, z). Thus, in general, problem (1) is an infinite-dimensional optimization problem, because the unknowns, (u, w), are functions defined over the domain Ω, though we avoid a formal discussion of function spaces in this paper.

We are working on a survey paper that reviews existing approaches to solve these problems, and highlights their computational and mathematical challenges. We also started to collect a set of benchmark problems for this challenging class of problems, and present some early numerical experience using both mixed-integer nonlinear solvers and nonlinear rounding heuristics.

Period at Host University. Universität Magdeburg, October 11–16, 2015. During the visit, we participated in the following activities:

- Research colloquium on "Mixed Integer PDE-Constrained Optimization" on October 15, 2015.
- Lecture for optimization Masters students on "Nonlinear Optimization Applications" on October 13, 2015. The lecture highlighted application of nonlinear optimization from a wide range of applications of interest to the Department of Energy, including power-grid design, control and operation; the design of complex structures such as nano-photonic devices; and the analysis of large-scale scientific data, including image analysis carried out at Argonne's Advanced Photon Source.

Period at Oberwolfach. Mixed-integer Nonlinear Optimization: A Hatchery for Modern Mathematics, October 18–23, 2015.

Follow-Up Activities. Following the research visit, we have already arranged for a number of follow-up activities with my host university, and with my host, Professor Sebastian Sager. The following research activities are under way or planned.

- A research internship by a student from the Universität Magdeburg (Anna Thuenen) through the end of December 2015. Anna is working on mixed-integer optimization problems arising in the control of the heat equation.
- Participation in a planning meeting for the optimization program at the Statistical and Applied Mathematical Sciences Institute (SAMSI). We have established a working group on MIPDECO, and are planning a research workshop on MIPDECO, which is likely to take place during SAMSI's year on Optimization under Uncertainty in 2016-2017.

Acknowledgments. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. This work was also supported by the Office of Advanced Scientific Computing Research, Office of Science, U.S. Department of Energy, under Contract DE-AC02-06CH11357.

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November 14, 2015

It is a pleasure to thank the Simons Foundation and the MFO for the Simons Visiting Professorship, which enabled me to spend a week at the Johannes Gutenberg University in Mainz prior to the October MFO workshop on mathematical models and modeling.

David Rowe, my host and workshop co-organizer, has just retired as Professor of History of Mathematics at Mainz. We have worked together on *The Mathematical Intelligencer*, mostly by long distance, for many years, he as editor of its history ("Years Ago") column, and I in various capacities (currently, Editor in Chief). The week gave us valuable time to talk in depth about the issues facing the *Intelligencer* and other matters of joint interest (see below), and I gave a department colloquium on the 38-year-old quarterly's past, present and future. The discussion that followed my talk ranged over the issues all forms of academic publishing are confronting at this time of rapidly evolving communication technologies. The audience's ages spanned a wide range, from students to emeriti, and the diversity and thoughtfulness of their views and suggestions were especially helpful to me in thinking about the *Intelligencer*'s next steps.

The MFO workshop itself was intense, stimulating, and engaging. Consequently, David and I are organizing an issue of *The Mathematical Intelligencer* for early 2017 that will feature five essays based on workshop presentations that showcase the historical, conceptual and material range of models and modeling. I agreed to serve as the reporter for that workshop and will send the abstracts to MFO soon, so I will not describe them here. My own talk focussed on the ways in which classical models in mathematical crystallography are misleading researchers today, as we try to understand the growth, form and properties of aperiodic crystals.

David Rowe and I stayed at MFO for a second week, courtesy of the Research in Pairs program (in this case a quartet: we were joined by Volker Remmert and David Peifer). Our goal for that week was to draft a propsal for a mini-workshop on the life and work of Max Dehn (1878 - 1952), who interests each of us deeply but in different ways. The mini-workshop will be a step toward assembling a collective biography of this remarkable, unusual, many-faceted mathematician. Through our discussions and research in the excellent MFO library, we found many new untapped resources. We have already submitted our RiP report, and hope to submit the miniworkshop proposal by May 1, 2016.

Again, I am most grateful for the Simons award. The in-depth time with David Rowe was most valulable and it was a pleasure to meet his colleagues and students and absorb something of the university and departmental atmosphere. The week in Mainz gave me perspectives that proved useful throughout my two-week stay at MFO, and will help shape the projects we discussed there.

Thank you!

Marjorie

Marjorie Senechal
Louise Wolff Kahn Professor Emerita in Mathematics and History of Science and Technology
Editor-in-Chief, *The Mathematical Intelligencer*
Fellow of the American Mathematical Society

DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCES
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Dear Sir or Madam:

This document describes my research and outreach activities during my participation to the MFO workshop on complexity theory and my subsequent visit to the University of Rome – Tor Vergata. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

On November 15-20, 2015, I was at the MFO to participate in the meeting on complexity theory. One of my goal in attending the meeting was to learn about a recently announced result of Eshan Chattopadhyay and David Zuckerman on constructions of two-source extractors and Ramsey graphs, applying recent results of Xin Li and Gil Cohen. Eshan, David, Xin and Gil were all among the meeting participants and they (along with Raghu Meka, who talked about follow-up work) all gave talks addressing different aspects of these new developments.

I was also interested in learning about certain recent results concerning the power of Sum-of-squares relaxations of combinatorial problems, and Boaz Barak announced at the meeting a proof of new results in this direction.

On November 21st I traveled to Rome, where I stayed until November 29. The main goal of my stay in Rome was to continue a collaboration with a group led by Andrea Clementi at the University of Rome – Tor Vergata. Clementi's group is interested in problems in distributed computing, and, more specifically, on the power of distributed algorithms in which every node has very limited space and, at each time step, performs only very simple computations. In the past, I collaborated with their group on the study of the convergence of algorithms to reach consensus in such model, and our current project is the study of graph-partitioning and community-detection algorithms in this model.

During my previous visit to Tor Vergata University, in May 2015, we initiated the investigation of such an algorithm, and we employed spectral techniques to analyze the algorithm. An initial report of this study is available at http://arxiv.org/abs/1511.03927. The results that we proved in May suffered from a number of shortcomings: they applied to a community-detection problem with two communities, while it is desirable to consider more general models with multiple communities; they suffered from an O(log n) loss in the conditions under which the distributed algorithm, compared to known centralized algorithms; and it required certain technical conditions that are not needed for centralized algorithms, and which we believe were not necessary.

During my November 21-29 visit, we discussed all of the issues above, and made some progress both on the question of working with more than two communities and on the technical issues needed to avoid the O(log n) loss.

One of the students of prof. Clementi's group (Emanuele Natale) is now visiting Berkeley during the current Spring semester, and we are going to formalize the proofs that we sketched in November and to write a revised preprint, which we plan to submit to one of the premiere conferences in the area.

On November 24, prof. Clementi organized in Rome an event titled "Kolmogorov meets Turing" with a series of talks on applications of probabilistic methods in computer science. I spoke of some recent work of mine on approximation algorithms for degree-bounded constraint satisfaction problems. Stefano Leonardi spoke about algorithms for Bayesian auctions, Francesco Pasquale spoke of past work, in which I was involved, on distributed algorithms to reach consensus, and Fabio Martinelli spoke of a Markov-Chain Monte-Carlo algorithm for counting triangulations. I was particularly interested in Martinelli's talk: he spoke of how to prove a certain log-Sobolev inequality, and use it to bound the mixing time of a Markov chain, and this is a style of argument that I have been trying to familiarize myself with, for different applications.

Sincerely

Luca Trevisan
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Scientific Activity Report: Surface PDEs with Willmore Flow

Robert Strehl

December 21, 2015

MFO Workshop: Geometric Partial Differential Equations: Surface and Bulk Processes (ID 1549)
Workshop Date: 29.11.-05.12.2015
Host: Prof. Dr. Stefan Turek, TU Dortmund University
Duration at Host University: 07.12.-18.12.2015

Together with my senior collaborator Dr. Andriy Sokolov (TU Dortmund University) we are working on the integration of Willmore energy in our numerical framework for surface and bulk partial differential equations (PDEs) arising in the field of biophysical processes. Hereby the surface undergoes deformations and evolves in time. In preceding papers, Sokolov et al. in Discrete and Continuous Dynamical Systems - Series B (2013) and Sokolov et al. in J. Comp. Appl. Math. (2015), the basis for the current research focus has been set. Therein the authors studied a numerically stabilized finite element (FE) scheme for PDEs on (evolving) surfaces. Preliminarily the evolution was prescribed by an explicit function for validation purposes. The aim is to allow for more complex and even implicit evolutions of the surface. Of particular interests are surface evolutions that minimize energies, such as the Willmore energy, or solution-driven evolving surfaces (MFO talk of Lubich et al., paper/preprint to appear). The practical relevance of non-explicitly given surface evolutions is out of question. In this regard we are currently considering biophysical processes that shape cell membranes or tissue development.

The goal of the research visit was to prepare a FE framework to integrate energy functionals that drive the evolution of surfaces. We successfully validated our implementation of Willmore flow in FEATFLOW following the guidelines in Droske and Rumpf, Interfaces and Free Boundaries (2004). Therein the authors proposed a reformulation of the level set evolution equation under Willmore flow. This results in a monolithic approach for the level set – curvature system that will be used as a basis to incorporate surface tension and volume preservation, two key-players in many relevant applications. Furthermore we aim to couple the Willmore flow solver with surface and bulk PDEs where the surface velocity is not explicitly prescribed but rather solution-dependent.

Stimulating discussions with participants of the MFO workshop shaped the outcome of this research visit and were utmost valuable to direct the ongoing collaboration with Turek and Sokolov. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Simons Visiting Professorship - Scientific Activity Report

Elisabeth M. Werner

This Simons Visiting Professorship Activity Report concerns the following two events: first, my participation at the workshop "Convex Geometry and its Applications" which took place at the MF Oberwolfach, December 6 - December 12, 2015. Second, my stay at the TU Wien, with Professor Monika Ludwig as my host, from December 14 - December 23, 2015.

These research stays were partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

At the Oberwolfach workshop I gave a talk "The Floating Body in Real Space Forms", based on recent work together with my post doc, Florian Besau [2]. Floating bodies had been defined earlier for convex bodies (convex compact subsets K with non-empty interior) in the Euclidean setting. For $\delta > 0$ small enough, the floating body K_δ is the intersection of all half spaces whose defining hyperplanes cut of a set of volume δ from K [9]. The convex floating body has played a desicive role in extending an important notion of affine convex geometry, the affine surface area, from smooth bodies to all convex bodies. It is imperative to have such extensions due to the fact that affine surface area is a powerful tool in the affine geometry of convex bodies and appears in applications ranging from PDEs to affine analytical isoperimetric inequalities, and to the approximation of convex bodies by polytopes.

In [2] we introduce the floating bodies, more generally, for spaces of constant curvature. Those admit a natural and intrinsic definition for floating bodies similar to Euclidean space. Our considerations lead to a seminal new surface area measure for convex bodies, which we call the floating area. This floating area is intrinsic to the constant curvature space and not only coincides with affine surface area in the flat case, but also has similar properties in the general case. Namely, the floating area is a valuation and upper semi-continuous. However, the group of transformations that leave it invariant is inherited from the space of constant curvature, in which it is intrinsic.

We draw a complete picture of this new notion of floating bodies and the floating area related to them in constant curvature spaces. An emphasis is put on hyperbolic space. For all proofs and more details we refer to [2].

The workshop at Oberwolfach provided a good opportunity to interact with many of my colleagues. With several among them I have joint projects and we were able to discuss those. So for instance with Grigoris Paouris (Texas A&M University) and Galyna Livshyts (Georgia Tech University).

At the TU Wien I gave a talk "Approximation of convex bodies by polytopes". In the talk I reported on recent results based on work with my PhD student Steven Hoehner and C. Schütt [4]. We investigate approximation of convex bodies by polytopes with a fixed number of vertices. While there is extensive literature on approximation of convex bodies by inscribed or circumscribed polytopes, much less is known in the case of generally positioned polytopes. We give upper and lower bounds for approximation of convex bodies by arbitrarily positioned polytopes with a fixed number of vertices in the symmetric surface area deviation. I refer to [4] for definitions and details.

As mentioned above, intertwined with the notion of floating body, is the notion of affine surface area. While I was at the TU Wien, Florian Besau, Monika Ludwig and I started a new project. We noticed that ideas from Florian Besau's and my work can be extended to a unifying approach. It seems that with this new approach we will be able to not only treat simultaneously previously obtained results about affine surface area in the Euclidean setting [9], the spherical setting [1] and the hyperbolic setting [2], or, more generally, the spaces of constant curvature setting, but also corresponding results for L_p affine surface area (e.g., [6, 10]), a generalization of affine surface area, which is at the core of the rapidly developing L_p Brunn Minkowski theory (e.g., [3, 5, 7, 8, 11, 12]).

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Simons Visiting Professorship - Scientific Activity Report

Ehud de Shalit

December 28, 2015

During the week of December 6-13 2015 I was invited to the University of Münster as a Simons Visiting Professor. My host was Prof. Peter Schneider. Subsequently, in the week of December 13-19, I participated in the workshop "Non-Archimedean Geometry and Applications" at Oberwolfach. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. I am grateful to both institutions for their support and for the conductive environment which they have provided.

I am currently engaged in two research topics. In the first, which is an on-going collaboration with Eyal Goren from McGill University, we study the geometry and arithmetic of Picard modular surfaces at a prime p which is inert in the underlying quadratic imaginary field. Although a lot of progress has been made recently on the geometry of unitary Shimura varieties, some aspects which are well understood today in the context of modular curves remain open in general, and our goal is to focus on the 2-dimensional example of Picard modular surfaces (associated with the group $U(2,1)$ and a quadratic imaginary field E), which is one of the simplest higher-dimensional examples. In particular, the geometry modulo p and the theory of p-adic modular forms on such surfaces is not fully understood when the prime p is inert in E (the split case being simpler). In a recent work we defined a certain differential operator denoted Θ on mod-p and p-adic modular forms, which on Fourier-Jacobi expansions has the effect of a "Tate twist". It is defined geometrically on a dense open subset of the Picard surface modulo p (the so-called μ-ordinary stratum) and we prove that it extends holomorphically across the supersingular strata. This Θ-operator resembles an operator which had been studied, in the context of modular curves, by Serre, Swinnerton-Dyer, Katz, Coleman and others, and proved instrumental for the study of p-adic modular forms and p-adic Galois representations attached to them. It has been generalized to Picard modular surfaces (and more general unitary Shimura varieties) at a split prime p by Ellen Eischen, but the case of an inert prime presents some new difficulties. A related result is the construction of a Hasse invariant in this context, and of a secondary Hasse invariant defined only on the super-singular locus (both have been constructed recently by other authors in much greater generality). The study of these Hasse invariants, with some intersection theory on the surface, allowed us to get a formula for the number of irreducible components of the super-singular locus on the Picard surface, in terms of its second Chern class. Currently, we are studying the structure of Picard modular surfaces with an Iwahori level structure at p.

I have given a one-hour talk entitled “A theta operator for Picard modular forms at an inert prime” about this topic in Münster.

The second research topic is unrelated, and concerns some open questions in elementary p-adic functional analysis. Let F be a local field of residual characteristic p and C an algebraically closed field of characteristic 0. Let $C_c^\infty(F)$ denote the space of C-valued locally constant compactly supported functions on F. Fix a non-trivial additive character on F, under which F becomes its own dual. Then Fourier transform is (almost) an involution on $C_c^\infty(F)$. When the field C is either $\mathbb{C}$ or $\mathbb{C}_p$, equipped with the sup norm, it is well-known that the Fourier transform is not well-behaved. For example, Fresnel and de-Matan have shown, in the 1970's, that there is a sequence of functions $\phi_n \in C_c(F/\mathcal{O}_F) \subset C_c^\infty(F)$ which converge to a non-zero $f \in C_0(F/\mathcal{O}_F)$ such that $\hat{\phi}_n$ converge to 0. A related question is whether for every $\epsilon > 0$, every $\phi \in C_c^\infty(F)$ has a decomposition $\phi = \phi_1 + \phi_2$ with $||\phi_1|| < \epsilon$ and $||\hat{\phi_2}|| < \epsilon$. The analogous question over $\mathbb{C}$ has a negative answer, but we suspect that over $\mathbb{C}_p$ this might be the case. Together with my student Amit Ophir we made some progress on this question, but it seems to be difficult, despite its elementary nature. A related question is whether an analogue of the Stone-von-Neumann theorem (on the irrdeucibility of the Heisenberg representation) holds true for the p-adic Banach space $C_0(F)$.

In my stay at Münster I discussed these questions with Peter Schneider, who is an expert on non-archimedean functional analysis. These discussions have not lead yet to the solution of the problem, but were very helpful. In particular, he told me of related old work of van-der-Put of which I was unaware.

The topic of the workshop in Oberwolfach was of great interest to me, although I do not currently work on questions directly related to it. I have learned a lot, and benefitted a lot from discussions with other participants. I am currently conducting (with David Kazhdan) a seminar on Scholze's work on perfectoid spaces and their arithmetic applications. Some of the talks in Oberwolfach, in particular the talks related to p-adic Hodge theory, were very helpful in this respect.

I want to thank the Simons Foundation, the Mathematisches Forschungsinstitut Oberwolfach and Prof. Peter Schnieder for their support and hospitality.

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