Mathematisches
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Simons Visiting Professors

Activity Reports 2016

Simons Visiting Professor Activity Report

David B. Leep

February 18, 2016

This is a report on my participation in the Simons Visiting Professor Program associated with my visit to Mathematisches Forschungsinstitut, Oberwolfach.

I visited Professor Detlev Hoffmann at the Technische Universitaet in Dortmund from January 17 - 23, 2016. I gave a seminar talk on Thursday, January 21. The title of the talk was "Artin's conjecture for diagonal forms over p-adic fields". I worked with Professor Hoffmann on our joint paper "Sums of Squares in nonreal Commutative Rings". This paper deals with the question of computing the level and Pythagoras number of an arbitrary commutative ring. We also introduced two other levels called the meta-level and hyper-level. We have obtained a lot of interesting results relating these levels with the Pythagoras number, including many results for generic rings that arise in our investigations. Our manuscript is nearly complete. In addition we started discussing a second paper extending some of the results of our paper.

I visited Professor Karim Becher at the University of Antwerp from January 23 - 30, 2016. In Antwerp, I worked with Professor Becher and held extensive mathematical discussions with two of Professor Becher's Ph.D. students, Parul Gupta and Sten Veraa. Parul Gupta is studying various local-global problems for quadratic forms. Sten Veraa has been studying extensions of Heath-Brown's theorem on solving systems of quadratic forms over a p-adic field. In both cases, I have been able to use my experience in these areas to offer advice to these students. I had discussions with Professor Becher on problems related to linkage and multiple linkage of quaternion algebras over fields.

I attended the Oberwolfach Workshop "Algebraic Cobordism and Projective Homogeneous Varieties" (#1605) from January 31 – February 6, 2016. As expected, there were five days of well-prepared talks on a variety of topics presenting the latest research of the various researchers. In addition, I continued my discussions with Professor Hoffmann and Professor Becher on our various research projects. The stimulating atmosphere of Mathematisches Forschungsinstitut, Oberwolfach always brings me great joy. This conference center is a real gem.

I am very grateful to the Mathematisches Forschungsinstitut Oberwolfach and the Simons Foundation for the existence of the Simons Visiting Professor Program, which allowed this three-week long visit to Dortmund, Antwerp, and Oberwolfach to take place.

This research stay was partially supported by the Simons Foundation and by Mathematisches Forschungsinstitut Oberwolfach.

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Scientific Activity Report

1. Research interests. My research interest is in complex geometry and representation theory of infinite-dimensional Lie algebras. More precisely, I am investigating the integrability in Gromov-Witten (GW) theory. The topological recursion is interesting to me, because it allows us to set up a recursion that computes the higher-genus GW invariants. The initial data of the topological recursion is the so-called spectral curve, which is a branched covering of the projective line with only simple ramifications. The recursion amounts to summing up certain residues at the ramification points. Recently, there was a work of V. Bouchard and B. Eynard, in which they identified the sum of the residues in the topological recursion with the contour integral of a certain meromorphic form on the spectral curve. In particular, by degenerating the spectral curve we can obtain more general recursions, where the ramifications points are not necessarily simple. On the other hand, in collaboration with B. Bakalov, I have developed a certain vertex algebra approach to GW invariants. Our construction amounts to writing a recursion for the GW invariants in terms of a certain vertex algebra, which in the physics literature is known as W-algebra. The problem is that the W-algebra is very difficult to work with, because it is defined as the kernel of certain screening operators, which in general is very difficult to compute. My idea is to compare the Bouchard-Eynard recursion with the W-algebra constraints. It is very natural to assume that they are the same. In particular, the Bouchard-Eynard recursion should give us a method to find solutions to the equations defined by the screening operators. I find this possibility very exciting.

2. Scientific activity at Oberwolfach. I have participated in the Oberwolfach workshop "Topological recursion and TQFTs", Feb 14-19, 2016. <u>I gave a lecture on "Vertex algebra recursion for the total descendant potential of A_N singularity"</u>. In this lecture I described my progress on comparing the Bouchard-Eynard recursion for A_N singularity with the vertex algebra approach mentioned in the above paragraph. I described certain states in the lattice vertex algebra corresponding to the Milnor lattice of the A_N singularity, which correspond to the Bouchard-Eynard recursion. <u>I also had many interesting discussions with N. Orantin and P.-D. Barkowski.</u> They have also established a Bouchard-Eynard recursion for A_N singularity, so they explained me in great details their approach.

3. Scientific activity at my host university. I visited Sergey Shadrin at the University of Amsterdam in the period Feb 20-27, 2016. <u>I gave a lecture at the student seminar on "Fano orbifold lines and integrable hierarchies" on Feb 23, 2016</u>. This was a 2 hours talk and one of my goals was to explain my work with B. Bakalov on vertex algebras in GW theory and how it can be used to construct integrable hierarchies. <u>During my visit I met every day with Sergey Shadrin and Guido Carlet.</u> I explained them my idea that the Bouchard-Eynard recursion should produce states in the W-algebra. We have tried to apply my ideas to the so-called boundary singularities. More precisely, we discussed in great detail the B_2 singularity. There is a general construction of integrable hierarchies due to

B. Dubrovin and Y. Zhang, which works also in the setting of boundary singularities. Yet another method to construct integrable hierarchies is based on the theory of simple Lie algebras and the resulting hierarchies are known as Kac-Wakimoto hierachies or Drinfeld-Sokolov. It is natural to expect that for boundary singularities, the Dubrovin and Zhang's construction is equivalent to Kac-Wakimoto or Drinfeld-Sokolov. This however is wrong and many specialists in integrable hierarchies find this failure very puzzling. That is why it is very interesting to compare the Bouchard-Eynard recursion with the vertex algebra approach. Unfortunately, we ran out of time. We were able to find the correct spectral curve, but it will take more time to complete the computation. I suspect that if successful this will lead to a very interesting paper. We also discussed certain integrable hierarchies in the work of S. Barannikov. This is related to an observation of Van de Leur, which evolved later on to a joint project with S. Shadrin, to classify semi-simple Frobenius manifolds via the multi-component KP hierarchy. Again, this is a far-reaching project, so the main outcome for me was that I was able to identify the mathematical problems that one should work on in the future. During my visit in Amsterdam, especially in the evenings, I continued to work on the topic that I presented at Oberwolfach. My expectation was that the states in the lattice vertex algebra mentioned in section 2 above are in fact in the W-algebra. I reduced the proof to a certain combinatorial identity. D. Lewanski, who is one of Shadrin's students, was very interested in the combinatorial identity. I had several discussions with him and as a result, in a couple of days he managed to find a proof. I wrote a paper in which D. Lewanski wrote an appendix with his argument (see arXiv: 1603.00073). The paper will be submitted soon to a journal.

4. Acknowledgement. The research stay reported in section 3 was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

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March 2, 2016

Professor Dr. Gerhard Huisken, Director
Mathematisches Forschungsinstitut Oberwolfach
Schwarzwaldstr. 9-11
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Dear Professor Dr. Huisken,

This letter is to acknowledge my greatest gratitude to you for your kind invitation to MFO in February of 2016. At the same time, this letter serves as a **Scientific Activity Report** of my stay at MFO and MPIM-Bonn, as a participant of the *Simons Visiting Professorship*.

The periods I spent as a Simons Visiting Professor in Germany are the following:

- MPIM-Bonn: From February 1 to February 14, 2016.
- MFO: From February 14 to February 20, 2016, participating in the MFO Workshop, 1607a, **Topological Recursion and TQFTs**.

During my stay at MPIM-Bonn, I had informal seminars with the following Guest Members of the Institute on various topics.

- Dr. John Alexander Cruz Morales: Cluster-algebra type mutations and Fano-variety/Landau-Ginzburg model mirror symmetry.
- Dr. Olivia Mirela Dumitrescu: Beilinson-Drinfeld opers, quantum curves, and variation of Hodge structures. A joint paper is planned from the discussions I have had with Dr. Dumitrescu.
- Dr. Petr Dunin-Barkowski: The Gaiotto Conjecture, quantum curves, and topological recursion.
- Professor Dr. Vasilisa Shramchenko of Sherbrooke University, Canada: Topological recursion for Landau-Ginzburg models.

At MFO, I gave a talk titled "*A solution to the Gaiotto Conjecture, and quantum curves*." I was also engaged in scientific discussions with the following participants.

- Dr. Murad Alim of Harvard University: Variation of Hodge structures, Hodge decompositions, and tt*-structures.

- Professor Dr. Jørgen Andersen of Aarhus University: Quantization of Hitchin moduli spaces.
- Dr. Olivia Mirela Dumitrescu of MPIM-Bonn: A new formulation of cohomological field theories and its applications. Another joint paper is planned from this discussion.
- Dr. François Petit of the University of Luxembourg: Rees D-modules, deformation quantization modules, and opers, globally defined on projective algebraic curves.
- Professor Dr. Jörg Teschner of the University of Hamburg: Quantization of Hitchin moduli spaces, quantum curves, the equivalence between quantum Garnier systems and BPZ equations, and equivariant cohomology of Hilbert schemes.
- Professor Dr. Katrin Wendland of the University of Freiburg: Mirror symmetry of Calabi-Yau spaces vs. Fano variety/Landau-Ginzburg duality, conformal blocks, conformal field theory, and variation of Hodge structures.
- Professor Dr. Don Zagier, Director of MPIM-Bonn: Possible relations between the roles of peculiar SL(2) in his current research projects, and the quantizability condition for Higgs bundles in which the SL(2) also plays an essential role in my current research.

Although joint papers are unlikely at this moment from these discussions, except for my current collaborator Dr. Dumitrescu, all the discussions mentioned above have been extremely useful for my research projects, and will be acknowledged in my forthcoming publications.

At the MFO Workshop 1607a, I presented my most recent work on solving the Gaiotto Conjecture with my collaborators Olivia Dumitrescu, Laura Fredrickson (University of Texas, Austin), Georgios Kydonakis (University of Illinois Urbana-Champaign), Rafe Mazzeo (Stanford University), and Andrew Neitzke (University of Texas, Austin). Physicist Gaiotto conjectured in his seminal paper titled "*Opers and TBA*" of 2014 a surprising construction of Beilinson-Drinfeld opers (globally defined Schrödinger-type equations on a compact Riemann surface) from a particular choice of families of Higgs bundles that are called *Hitchin components*. Through the joint work with Dumitrescu, we are able to give a purely holomorphic description of the Gaiotto opers, which are constructed by scaling limit of real differentiable objects. Our point of view makes it clear that the whole business is deeply related to variation of Hodge structures, and this very point triggered lively discussions with many of the participants of the MFO Workshop. I was also able to learn different physical vision on the same subjects from Professors Alim, Teschner, and Wendland. It was a total surprise to me that seemingly the same SL(2) that plays an essential role in my quantization process is also appearing in Zagier's current research topics (yet to be published).

The opportunity to engage in numerous timely discussions with the experts in both mathematics and physics on a brand-new important result—this is very rare occasion, and I am immensely grateful for MFO to provide such a fantastic place to facilitate the lively interactions. Dr. Dumitrescu and I particularly liked the little discussion space with a chalkboard at the end of the library. What we thought as "Die mathematische Forschung Geist der schönen Ort" affected our own spirits, so that she and I were able to make a new discovery that we did not expect at all in that library space. Still a lot of work has to be done, but we were startled to notice the new dimensions that we had long thought impossible.

My visit was en exceptionally happy experience for my mathematical research. Thank you very much for your leadership and vision of the Institute!

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Sincerely yours,

Motohico Mulase
Professor of Mathematics

Scientific Activity Report

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1. Timetable

February 21–26	Participation in the Oberwolfach Workshop *New Developments in Functional and Highly Multivariate Statistical Methodology*
February 29–March 11	Guest of Prof. Dr. Holger Dette at Ruhr-Universität Bochum
February 25	Presentation on *Dating structural breaks in functional data without dimension reduction* at the Oberwolfach Workshop
March 4	Seminar on *Dating structural breaks in functional data without dimension reduction* at Université catholique de Louvain in Louvain-la-Neuve, Belgium

2. Scientific activities

During this research stay two research projects were investigated, both related to the theme of the workshop and in collaboration with the host of this research stay, Prof. Dr. Holger Dette. The first project is on variable selection in fully functional linear models and the second project on the detection of relevant structural breaks in functional data. A brief summary of the activities is given in the following.

(a) Variable selection in fully functional linear models.

Functional data analysis has emerged as a meaningful inferential tool for some of the contemporary data that often exhibit complexity in size and structure, with functional regression models playing the most important role. Within the class of functional regressions, those with functional predictors and scalar responses have seen a larger share of research contributions. In some cases, however, it may be more appropriate to entertain fully functional regression models for which both response and predictors consist of functions. This may, for example, be the case for the monitoring of daily particulate matter concentration profiles at a certain measuring station. It is known that these concentrations may correlate with temperature profiles taken at different altitudes, pollution levels measured at nearby stations and other measurements well represented by functions. For the model building process it is then important to identify the significant ones among the set of potential predictors.

Variable selection methods have gained increasing attention because of the emergence of complex and big data for which the traditional stepwise methods may be computationally prohibitive. New algorithms based on a regularization framework have been developed, chief among them the LASSO and the SCAD penalties, and their many extensions. Variable selection for functional linear models has so far been discussed only in the context of predictor functions and scalar responses. Most of the existing literature deals with employing versions of the LASSO regularization technique after a representation of the predictor functions in a certain basis function system. There is, however, no work studying variable selection in the fully functional regression setting as of now. Somewhat unrealistically, most contributions to the area utilize only a single predictor function for a functional response, so that the problem of selecting variables does not occur. The main goal of this project is therefore to study fully functional regression models with multiple predictor functions, and to develop statistical methodology that aids extraction of significant predictors from the full model. It is currently explored to achieve this goal by first performing a functional principal components (FPC) analysis on both response and predictor functions, thereby creating an auxiliary multivariate linear model consisting of FPC score vectors and matrix representations of the functional regression operators in the eigenfunction bases of the response and corresponding predictors. Then, select variables in the auxiliary multivariate regression. The multivariate predictors can be turned into functional predictors by means of a truncated Karhunen–Loève expansion. Since there is a neat decomposition of the functional prediction error into a multivariate component and a component determined by the complement of the space spanned by the first few response eigenfunctions, a functional variable selection criterion can be introduced. The criterion selects in particular the appropriate number of response and predictor FPC scores automatically and does not rely on ad-hoc procedures.

(b) Detection of relevant structural breaks in functional data.

Structural break detection is an area of interest for researchers from diverse backgrounds, including statistics, economics and mathematics. Particular interest for this project is in defining the notion of relevant structural breaks for functional data and to initiate a new research direction within the field. Relevant structural breaks allow for small but quantifiable deviations from the traditionally employed sharp null hypothesis. This extension comes with significantly increased mathematical complexity. For functional data, this leads to the derivation of theoretical results in Hilbert spaces and Banach spaces.

The focus is first on detecting relevant structural breaks in the mean function. Here, from a statistical point of view, the choice of the norm in which differences in the mean functions are measured becomes important. The methodology to be developed should include a spectrum of choices, as different applications may lead to different norm selections. Associated with this starting point are a wealth of both applied and theoretical questions to be explored in the future.

3. Acknowledgement

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

SCIENTIFIC ACTIVITY REPORT (SVP)

SHIGEYUKI KONDŌ

Periods of the stay

Oberwolfach: 24 April, 2016 – 29 April, 2016.
Amsterdam: 29 April, 2016 – 12 May, 2016.

Research

I am now interested in Enriques surfaces in characteristic 2. An algebraic surface over the complex number is called an Enriques surface if its universal cover (in fact, it is a double cover) is a $K3$ surface. We have a strong result called Torelli type theorem for $K3$ surfaces (due to Piatetski-Shapiro and Shafarevich) which can be applied to study Enriques surfaces. As a result Enriques surfaces over the complex numbers have been studied well. On the other hand, Enriques surfaces in positive characteristic, especially in characteristic 2, are still mysterious. Bombieri and Mumford classified Enriques surfaces in characteristic 2 into three classes called "classical", "singular" and "supersingular" Enriques surfaces. As in the case of characteristic 0, there exists a canonical cover of any Enriques surface, but in this case it might be non-normal. One of the most interesting problems is to classify all Enriques surfaces in characteristic 2 with finite automorphism groups. The most difficult cases to deal with are the classical and supersingular Enriques surfaces whose canonical cover are non-normal.

In this stay in Oberwolfach and at the University of Amsterdam, I have found a concrete construction of a supersingular Enriques surface with a finite automorphism group whose canonical cover is non-normal. I think that this research will bring us to the next step of classification of Enriques surfaces in characteristic 2 with finite automorphism groups.

Lectures

During my stay I gave three talks, namely the following:

1) 24 April, 2016: Enriques surfaces in characteristic 2 with finite automorphism groups, Oberwolfach workshop "Moduli spaces and Modular forms" (24 April - 30 April 2016).

2) 10 May, 2016: The symmetry group of degree 6 and special Enriques surfaces in characteristic 2, GAGA Seminar at Utrecht University.

3) 11 May, 2016: The Leech roots and some $K3$ surfaces, Algebraic Geometry Seminar at the University of Amsterdam.

Discussions with other mathematicians

In Oberwolfach, I had the opportunity to discuss with many mathematicians. In particular I discussed with T. Katsura, C. Liedtke, V. Nikulin, M. Schütt, N. Shepherd-Barron on Enriques surfaces, and with J. Bruinier, V. Gritsenko, K. Hulek, R. Laza, S. Ma, R. Salvatti-Manni, K. Yoshikawa on moduli spaces and modular forms.

In the Amsterdam, I discussed with Professor van der Geer on the moduli space of Enriques surfaces in characteristic 2, and on the arithmetic ball quotients which are moduli spaces of some $K3$ surfaces.

Acknowledgement. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

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Scientific Report

Alina Chertock

Period spent at Stuttgart University: June 8, 2016–June 12, 2016.

Period spent at University of Mainz: June 12, 2016–June 19, 2016.

Period spent at Oberwolfach and the Oberwolfach Workshop: June 19, 2016–June 25, 2016.

During the reported period, I have been working on developing asymptotic preserving (AP) numerical methods in the context of two different problems — multi-phase flow models and multiscale models for glioma spread. The first project has been carried out in collaboration with Prof. Christian Rhode (Stuttgart University), Dr. Jochen Neusser (Stuttgart University) and Prof. Pierre Degond (Imperial College). The second project has been carried out in collaboration with Prof. Christian Rhode (Stuttgart University) and Prof. Maria Lukacova (University of Mainz).

Multi-phase flow models There are two approaches to describe the behaviour of two-phase fluids, the sharp interface (SI) and the diffuse interface (DI) approach. The first approach describes the two phases with two different sets of equations that are coupled by some interface conditions. The second approach needs only one set of equations to model the two phases. As we don't have to track the location of the interface explicitly, the DI approach is used to describe, e.g. the merging process of droplets and bubbles.

We considered two DI models for a homogeneous compressible fluid: The Navier-Stokes-Korteweg system (NSK) and a relaxation system for the NSK system. The NSK equations are a classical diffuse-interface model for compressible two-phase flows. As direct numerical simulations based on the NSK system are quite expensive and in some cases even impossible, we consider a relaxation of the NSK system, for which robust numerical methods can be designed. However, time steps for explicit numerical schemes depend on the relaxation parameter and therefore numerical simulations in the relaxation limit are very inefficient. To overcome this restriction, we propose an implicit-explicit asymptotic-preserving finite volume method. We prove that the new scheme provides a consistent discretization of the NSK system in the relaxation limit and demonstrate that it is capable of accurately and efficiently computing numerical solutions of problems with realistic density ratios and small interfacial widths.

Multiscale models for glioma spread Cancer cell migration is an essential feature in the process of tumor spread and establishing of metastasis. It characterizes the invasion observed on the level of the cell population, but it is also tightly connected to the events taking place on the subcellular level. These are conditioning the motile and proliferative behavior of the cells, but are also influenced by it. In the current project, we consider a multiscale model linking the micro and macro levels and aiming to assess their interdependence. We study a two-dimensional multiscale haptotaxis model and propose a new asymptotic preserving (AP) numerical scheme that reflects the convergence of the studied micro-macro model to its macroscopic counterpart in the singular limit. The method is based on the operator splitting strategy and a suitable combination of the higher-order implicit and explicit time discretizations. In particular, we use the so-called odd-even decoupling and approximate the stiff terms arising in the singular limit implicitly. Our goal is to prove that the resulting scheme satisfies the AP property. More precisely, it yields a consistent approximation of the limiting system as the mean-free path converges to 0. The performance of the scheme will be illustrated on a number of numerical experiments.

Acknowledgement: This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Scientific Activity Report

Changlong Zhong

This is the scientific activity report about my stay at the Mathematisches Forschungsinstitut Oberwolfach (MFO) during the week of June 26th - July 2nd, and at the University of Duisburg-Essen during July 2nd-July 8th, as a Simons Visiting Professor.

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach, and also supported by Dr. Marc Levine at the University of Duisburg-Essen. I would like to thank them for the host and support.

During my stay at Oberwolfach, I attended the workshop "Algebraic K-theory and Motivic Cohomology", organized by Thomas Geisser, Annette Huber-Klawitter, Uwe Jannsen, and Marc Levine.

During my stay at Essen, I talked with Dr. Marc Levine several times. More frequently, I talked with his other visitor, Dr. Gufang Zhao, from University of Massachusetts at Amherst. Jointly with Dr. Zhao and Changjian Su from Columbia University, we worked on a project related to restriction formula of the K-theoretic stable basis of Springer resolutions, which was defined by Maulik and Okounkov [MO12, OK15, OS16]. Here the restriction means the restriction to the K-theory of the T-fixed points of the cotangent bundle T^*G/B where G/B is a complete flag variety.

We first use the language of formal affine Demazure algebra introduce in [CZZ16, CZZ13, CZZ15] to construct the stable basis algebraically, i.e., based only on information of the root system associated to G. More precisely, we define the stable basis by applying the basis T_w of Hecke algebra H on the class of identity point in $K_0^{T\times\mathbb{C}^*}(T^*G/B) \cong K_0^{T\times\mathbb{C}^*}(G/B)$. We then use the generalized root polynomials introduced in [LZ14] to compute the restriction formula of the algebraically-defined stable basis.

In the end, we define some operators coming from $K_0^{G\times\mathbb{C}^*}(Z)$, Z being the Springer variety. On one side, we show that these operators coincide with $T_w \in H$, and on the other side, we show that these operators also permute the geometrically-defined stable basis. Hence, our algebraical stable basis agree with the geometric one of Maulik-Okounkov.

This work is almost done and it will appear at arXiv pretty soon.

Surprisingly, we notice there is some close relationship with the coefficients studied in [NN15], or more precisely, they are equal up to certain normalization. In other words, there is some connection between K-theoretic basis and the natural basis of the unramified principal series representation in number theory. It will be our future work in studying this connection.

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Report: Simons Foundation Supported Visit
by Jon A. Wellner
to Lund University, Sweden, June 24 - July 1, 2016
Host: Dragi Anevski

Jon Wellner visited Dragi Anevski at Lund University (Sweden) during the period 24 June - 1 July 2016, just before the workshop at MFO on "Statistics for Shape and Geometric Features" during the period 3-9 July 2016. Dragi Anevski was one of the co-organizers for this workshop.

During the visit to Lund, Anevski and Wellner discussed a draft manuscript by Anevski, Richard Gill, and Stefan Zohren entitled "Estimating a probability mass function with unknown labels". This paper is closely related to the famous "missing species problem" which has been studied from numerous perspectives, and which is closely related to several of Wellner's ongoing research interests. These discussions also resulted in the framing of several key questions concerning rates of convergence in the model studied by Anevski, Gill and Zhoren, and lower bounds for estimation in this type of problem. The issue of finding matching upper and lower bounds seems worthy of further investigation.

Anevski and Wellner also discussed several problems concerning shape restricted inference including confidence bounds for several parameters of interest, including confidence bounds for the mode of a log-concave density. This is on-going work between Wellner and Charles Doss (University of Minnesota).

This research stay was partially supported by the Simons Foundation and by Mathematisches Forschungsinstitut Oberwolfach.

SCIENTIFIC ACTIVITY REPORT

AGNÈS BEAUDRY
JULY 29, 2016

- Host: Hans-Werner Henn, Université de Strasbourg
- Period: June 18 to July 16, 2016
- Oberwolfach Workshop: Topologie, July 17 to July 23 2016

Research Collaboration

My visit at the Université de Strasbourg was divided between two projects.

(1) The $K(2)$–local Picard group (joint with Bobkova, Goerss and Henn)
(2) The chromatic splitting conjecture (joint with Goerss and Henn)

Acknowledgement

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. Further, this material is based upon work supported by the National Science Foundation under Grant No. DMS-1612020.

Background

For each prime p, there are higher analogues of mod p and p–completed K–theory, called the Morava K-theories $K(n)$ and the Morava E–theories E_n. They have higher analogues of Bott Periodicity and, like the image of J, this gives rise to periodic families of elements in the stable homotopy groups of spheres. Understanding and computing these periodic families is closely tied to understanding the $K(n)$–localization $\mathcal{S}_{K(n)}$ of the homotopy category of spectra.

(1) The $K(2)$–local Picard group

The $K(n)$–local Picard group, Pic_n, is the group of isomorphism classes of invertible objects in $\mathcal{S}_{K(n)}$. It plays an important role in our understanding of the structure of $\mathcal{S}_{K(n)}$. For example, the elements of Pic_n give rise to automorphisms of $\mathcal{S}_{K(n)}$. Further, the $K(n)$–local category admits a certain kind of duality, called Brown-Commenetz duality, whose dualizing object has the form $L_{K(n)}(E_n \wedge I_n)$ for an element I_n of Pic_n. Therefore, computations of Pic_n play an important role in our understanding of the structure of $\mathcal{S}_{K(n)}$. The group Pic_n has been computed if $n = 1$ for all primes and if $n = 2$ for odd primes. An ongoing project is to compute it when $n = 2$ and $p = 2$.

There is an algebraic counterpart to Pic_n called Pic_n^{alg} and a map $\mathrm{Pic}_n \to \mathrm{Pic}_n^{alg}$. The kernel is denoted κ_n. Elements of κ_n are called *exotic*, as the difference between them and $L_{K(n)}S$ is purely topological, making them hard to detect. The group

Pic_n^{alg} is, in theory, more accessible than κ_n. The group Pic_2^{alg} at $p = 2$ has been computed by Hans-Werner Henn, and our current focus is to compute κ_2. During this visit, we finished the proof of the following result.

Proposition 0.1. *There is a filtration $\kappa_2^0 \subseteq \kappa_2^1 \subseteq \kappa_2$ such that*
- $\kappa_2^0 \cong \mathbb{Z}/4 \oplus \mathbb{Z}/8$
- $\kappa_2^1/\kappa_2^0 \cong \mathbb{Z}/2$

Further, we have a solid sketch for an argument that $\kappa_2/\kappa_2^1 \cong \mathbb{Z}/8$, but are still in the process of filling in details.

(2) The chromatic splitting conjecture

The Chromatic Splitting Conjecture is one of the most important structural conjectures in chromatic homotopy theory. It studies the relationship between the local categories $\mathcal{S}_{K(n)}$ as n varies. The main object of study in the conjecture is

$$L_{n-1}L_{K(n)}S = L_{K(0)\vee\ldots\vee K(n-1)}L_{K(n)}S.$$

The conjecture predicted a certain splitting for this spectrum. At $n = 2$, the statement was that

$$L_1L_{K(2)}S \simeq L_1S_p \vee L_1S_p^{-1} \vee L_0S_p^{-3} \vee L_0S_p^{-4}.$$

However, I disproved this statement when $p = 2$, and the goal of this joint project has been to compute the homotopy type of $L_1L_{K(2)}S$ at the prime 2. We have proved the following result:

Theorem 0.2. *Let $V(0)$ be the cofiber of $S \xrightarrow{2} S$. Then*

$$L_1L_{K(2)}S \simeq L_1S_2 \vee L_1S_2^{-1} \vee L_0S_2^{-3} \vee L_0S_2^{-4} \vee L_1\Sigma^{-2}V(0)_2 \vee L_1\Sigma^{-3}V(0)_2.$$

During my visit, we finished the proof and made significant progress toward writing the paper for this result

Colloquia, lectures and public outreach

During my stay, I gave two lectures. The first was in the Séminaire: Algèbre et Topologie at the Université of Strasbourg on June 28, 2016. The talk was entitled "The *bo* and *tmf* resolutions" and described a joint project with Behrens, Bhattacharya, Culver, Ravenel and Xu on using the *tmf*–resolution in order to study the Telescope conjecture at chromatic level $n = 2$. The second lecture was in the Séminaire de Topologie Algébrique at the Université de Paris 13 on July 4, 2016. The talk was entitled "The Chromatic Splitting Conjecture" and described the joint project with Goerss and Henn that was discussed above.

In terms of other public outreach, I also talked with Viet-Cuong Pham, Prof. Hans-Werner Henn master's student. We discussed his studies of the proof by Miller of the telescope conjecture at odd primes and chromatic level $n = 1$.

SCIENTIFIC ACTIVITY REPORT: 5 JULY – 22 JULY, 2016

EMILY RIEHL

Logistics

I visited Thomas Nikolaus at the Max Planck Institute in Bonn from 5 July – 17 July, 2016. I then attended the Topologie workshop at Oberwolfach from 17 July – 22 July, 2016. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Scientific Activities

Discussants. At the Max Planck Institute, I spent the majority of my time involved in mathematical discussions with Thomas Nikolaus and also Dominic Verity (from Macquarie University in Sydney, Australia) and David Gepner (from Purdue University in Purdue, IN, USA) who happened to be in town during that period. These conversations with Nikolaus and Gepner continued at Oberwolfach with Akhil Mathew (Harvard University, Cambridge, MA, USA), Justin Noel (University of Regensburg, Regensburg, Germany), and Nick Rozenblyum (University of Chicago, Chicago, IL, USA) also contributing.

The problem. The researchers mentioned above share a joint interest in what might be called *higher ∞-category theory*. Certain varieties of mathematical structures — chain complexes, higher stacks, n-manifolds — naturally inhabit weak infinite-dimensional categories. Such categories contain "k-dimensional morphisms" for each $k \geq 1$ that can be composed in an associative and unital manner, up to a weakly invertible morphism of one higher dimension. Frequently the infinite-dimensional categories that one encounters are (∞, n)-*categories* for some $n \geq 0$, meaning that once $k > n$ all k-morphisms are equivalences, in a suitable weak sense. For example the category of n-bordisms with corners is an (∞, n)-category, with diffeomorphisms and isotopies of such serving as the higher dimensional morphisms. By Grothendieck's "homotopy hypothesis," $(\infty, 0)$-categories are understood to be topological spaces. As n increases, these category notions become more general

$$\mathsf{Spaces} \simeq (\infty, 0)\text{-}\mathsf{Cat} \subset (\infty, 1)\text{-}\mathsf{Cat} \subset (\infty, 2)\text{-}\mathsf{Cat} \subset \cdots \subset (\infty, \infty)\text{-}\mathsf{Cat},$$

and their theory becomes considerably more complicated, among other reasons because ordinary weak n-category theory is not well understood for $n > 2$.

Following Lurie, the term ∞-*category* is a common nickname for $(\infty, 1)$-category, i.e., a category that is weakly enriched in topological spaces. Joyal, Lurie, and others (including Nikolaus, Gepner, and collaborators) have developed an extensive theory of ∞-categories. Verity and I have been working on a new "model-independent" approach to the foundations of ∞-category theory (see [RV1, RV2, RV3, RV4, RV5, RVx]). One of the highlights of Lurie's work, is his theory of *presentable ∞-categories*, a homotopical extension of the theory of locally-presentable categories, whose objects are generated under filtered colimits by a certain class of "compact" objects.

Earlier this year, Thomas Nikolaus began investigating an as-yet-undefined notion of *presentable higher ∞-category*, generalizing from $(\infty,1)$-categories to (∞,∞)-categories. His hope was that the theory of such higher ∞-categories could be developed using a particularly well-behaved model: *complicial sets* (nee. weak complicial sets), studied extensively by Verity. The majority of my stay at Bonn and Oberwolfach was spent thinking about this notion.

Progress. We investigated:

- A variety of ways to characterize the ∞-category of (∞,∞)-categories, in particular focusing on its relationship to the ∞-categories of (∞,n)-categories.
- An explicit model for the complicial set of (∞,∞)-categories, as the homotopy coherent nerve of the complicially-enriched category of saturated complicial sets.
- Potential "fibrational" and "cofibrational" characterizations of functors valued in (∞,∞)-categories.
- Higher dimensional analogs of the class of *filtered colimits*.

We also shared information about related topics that are better understood, including:

- The Gepner-Haugseng notion of enriched ∞-category.
- The lax gray tensor product for (∞,∞)-categories modeled in complicial sets.

The theory of higher ∞-categories is in its infancy, so these conversations were extremely valuable. Early stage collaborations are much more easily done in person. I plan to continue working in this area and I am certain that some of these investigations will eventually be written up in a paper, at which point I will gratefully acknowledge the Simons Foundation and the Mathematisches Forschungsinstitut Oberwolfach for supporting this visit.

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– Scientific Activity Report –

Heiko Dietrich

Simons Visiting Professorship at RWTH Aachen

As an invited speaker, I have attended the Oberwolfach Workshop "Computational Group Theory" ID 1631 from July 31 to August 6, 2016. Subsequently I travelled to the RWTH Aachen to take up the awarded Simons Visiting Professorship (SVP) hosted by Prof Gerhard Hiss and Dr Markus Kirschmer; I was a guest of the RWTH Aachen from August 6 to August 18, 2016.

Scientific report on my stay at Oberwolfach

This year's visit to the MFO was my third attendance of the workshop series "Computational Group Theory" (2006, 2011, 2016). As in previous years, I was impressed by the high quality of talks and the excellent research environment provided by the MFO. Being in one place with so many experts of my research discipline always is a unique and inspiring experience, and I had many exciting conversations with colleagues and collaborators. The following paragraphs list a few highlights.

Prof Bettina Eick and I have a joint DAAD-Go8 travel grant in 2015/16, which allows us to work on a project in Coclass Theory. During our MFO workshop participation, Prof Eick and I had the chance to continue with this research project; we also used the opportunity to revise a recently submitted research publication and to send the final version back to the editor. On the second day of the workshop, I was invited to give a presentation about our project to the workshop participants.

Together with Dr Tobias Moede and Prof Charles Leedham-Green, I worked on a related project (also in Coclass Theory); we discussed approaches how to deal with so-called non-skeleton groups in coclass graphs. We will continue this collaboration later this year when Dr Moede visits me in Melbourne.

With Prof Max Horn and Dr Frank Lübeck I have discussed the progress of the GAP software package Recog; this is an extensive project to translate existing implementations of Matrix Group Recognition algorithms from the computer algebra system Magma to the system GAP. Since I am one of the main authors of the Magma code for constructive recognition of classical matrix groups in even characteristic, these discussions were very useful. In the context of algorithms for matrix groups, I had also interesting conversations with Dr Alla Detinko. My previous focus in computational matrix group theory was on finite matrix groups; Dr Detinko is an expert for infinite matrix groups, and it was very inspiring to learn about the results in the so-called finite-by-solvable case. Prof Horn and I also considered some other GAP related issues, in particular, the GAP package AnuPQ which is co-maintained by Prof Horn.

A/Prof Willem de Graaf and I have continued to work on our project on classifying nilpotent orbits in so-called real θ-groups. Supported by computations, we worked out a number of new explicit examples. We also identified some theoretical obstacles we will have to work on in the future. One outcome of our discussions was the plan to submit a joint grant application in 2017 to the Australian Research Council.

A/Prof James Wilson gave an inspiring lecture on group isomorphisms; amongst others, he mentioned a 2005 paper of mine on so-called cubefree groups. His complexity analysis of my algorithm motivates me to revisit my algorithm and to devise a more efficient isomorphism test for cubefree groups.

Scientific report on my stay at the RWTH Aachen

The RWTH Aachen is one of the main centres of Computational Group Theory in Germany; it is also the birth place of the computer algebra system GAP (released 1988 by Prof Joachim Neubüser), which nowadays is one of the main international computer algebra systems. A number of leaders in my field are at the RWTH Aachen, for example, Profs Hiss, Nebe, Niemeyer and Drs Lübeck, Kirschmer, which is the main reason why I haven chosen the RWTH Aachen for my SVP; I mention a few highlights below.

I have continued discussions with Dr Lübeck about the `Recog` package. We have analysed the existing Magma implementation of my constructive recognition algorithms in detail, and we have developed strategies how to incorporate these algorithms in GAP. This will be a tedious and technical undertaking and it is very valuable that we had the chance to consider this matter in more detail.

I have two projects which involve computations with finite groups of Lie type: a classification project of radical subgroups (in finite exceptional groups of Lie type) and a project in computational Lie theory (involving semisimple algebraic groups). I have discussed the progress and current state of these projects with Prof Hiss, who is an expert in computing with groups of Lie type. During my stay I was writing a set of notes about constructing the so-called real Weyl group, based on papers of Adams & du Cloux and Vogan, which is of importance to my joint project with A/Prof de Graaf.

Dr Kirschmer has some expertise in computing with infinite matrix groups, in particular, with groups containing free groups as subgroups; his work is therefore complementary to the work of Dr Detinko mentioned above. Given my background in computing with finite matrix groups (and my conversations with Dr Detinko), I was very interested in Dr Kirschmer's work and we studied in detail his two recent papers on this topic. As a preparation, I had started to work on these papers on the weekend of my arrival.

Motivated by A/Prof Wilson's talk about isomorphism testing of cubefree groups, I have revisited my 2005 paper on cubefree groups. I have started to turn the construction algorithm of that paper into an algorithm to construct isomorphisms between cubefree groups. A partial implementation of the algorithm seems to perform very well; I have started to write down the theory for that algorithm and it is planned to extend these notes to a joint publication together with A/Prof Wilson.

Acknowledgements

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach; I thank both for their support and the opportunity to visit the RWTH Aachen. I also wish to thank the organisers of the workshop, Profs Bettina Eick, Gerhard Hiss, Derek Holt, and Eamonn O'Brien, for running such an interesting and successful workshop, and for nominating me to the SVP scheme. I am particularly grateful to my hosts Prof Gerhard Hiss and Dr Markus Kirschmer for their support and making me feel welcome at the RWTH Aachen.

Oberwolfach Simons Visiting Professor Scientific Activity Report

James Brannick *

Pennsylvania State University, Department of Mathematics

This article summarizes my activities during my visit to Oberwolfach and University of Wuppertal from September 3-17 as a Simons Visiting Professor. During the first week of my trip I attended the Oberwolfach Workshop:

"Self-Adaptive Numerical Methods for Computationally Challenging Problems"

The workshop took place at the Mathematisches Forschungsinstitut Oberwolfach from September 4-10 and was organized by R. Bank, Z. Cai, and R. Verfürth. Self-adaptive numerical methods provide a powerful and automatic approach in scientific computing, in particular, we note the Adaptive Mesh Refinement (AMR) algorithms that have been widely used in computational science. The key ingredient for success of self-adaptive numerical methods is a posteriori error estimates that are able to accurately locate sources of global and local error in the current approximation. The focus of this workshop was on the design and analysis of such estimators.

I attended the entire meeting, which included roughly 6 talks on adaptive methods each day. I note that adaptivity in mesh refinement is a topic that I recently started working on in connection with my research on bootstrap finite element methods for solving PDE eigenvalue problems, see e.g., the paper [3]. Scientifically, I benefited from hearing talks by leading experts on the latest developments in this important field. In addition, I developed a new collaboration on developing estimators for surface PDE eigenvalue problems with two of the participants in the workshop, both will visit our research center this upcoming year. To the best of my knowledge this is a topic for which little work has been done and so is of general interest, though it will directly impact our research on the Laplace-Beltrami eigen-problem as well.

After the workshop I traveled to Wuppertal, Germany to visit Andreas Frommer and his group at the University of Wuppertal. I visited Wuppertal from September 10-17 to collaborate on the development of multilevel methods for solving coupled systems of partial differential equations (PDEs) and elliptic eigenvalue problems. The weeklong visit to Wuppertal led to some unexpected

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and important new results in our research on the design and analysis of bootstrap multigrid [2] for PDE systems and algebraic eigenvalue problems. This new result concerns an optimal (two-level) form of algebraic multigrid interpolation that gives the best possible convergence rate for a coarse space of fixed dimension. Ultimately, it leads to a generalized eigenvalue problem involving the so-called symmetrized smoother that needs to be tied to the coarse space construction. These results we have in turn used to develop more practical and robust algebraic multigrid solvers that use general block smoothers. In our view, these are important new results concerning a topic we have been working on for numerous years [2, 5, 6, 1, 4]. We are currently writing a paper based on these results. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschunurn led to gsinstitut Oberwolfach.

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SCIENTIFIC REPORT FOR THE SIMONS VISITING PROFESSOR AT MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

JUN HU
SCHOOL OF MATHEMATICAL SCIENCES, PEKING UNIVERSITY

I would like to thank the institute for providing such an great offer to me. I also took this opportunity to thank my host Professor Carsten Carstensen from the Department of Mathematics at Humboldt University of Berlin for covering my travel fee between Mathematisches Forschungsinstitut Oberwolfach and Berlin, and my hotel fee (including wonderful meals and wines) in Berlin from September 10–September 18.

1. Scientific Activities

During my stay as a Simons Visiting Professor, I participated the workshop "Self-Adaptive Numerical Methods for Computationally Challenging Problems" organized by Randy Bank, Zhiqiang Cai, and Rüdiger Verfürth, September 4–September 10, Mathematisches Forschungsinstitut Oberwolfach, visited Professor Carsten Carstensen at Humboldt University of Berlin, September 10–September 18, delivered a talk entitled "Adaptive and Multilevel Mixed Finite Element Methods", September 14, at Department of Mathematics, Humboldt University of Berlin, and finally participated the workshop "Adaptive Algorithms" organized by Carsten Carstensen and Rob Stevenson, September 19–September 24, Mathematisches Forschungsinstitut Oberwolfach and presented a talk entitled " Adaptive and Multilevel Mixed Finite Element Methods". As culture excursions, I visited A. Einstein summer House in Berlin at September 11 and went to a wonderful concert performance (invited by my host Professor Carsten Carstensen) at night September 14.

2. Scientific Progress

2.1. **Convergence of the Adaptive $P4/P3$ element method for the Stokes Equations.** In the Stokes equation, there are two variables the velocity and the pressure, which form a saddle problem. Therefore, Galerkin-orthogonality or quasi-orthogonality for the positive and definite problem is lack for this type of problems. Moreover, the technique used to the nonconforming and mixed methods can not be used herein. Because of this reason, convergence and optimality was only analyzed and established for the adaptive nonconforming linear element so far. In 2006, in a preprint, Hu and Xu proposed an idea to separate the convergence of the velocity from the pressure and consequently first establish the convergence of the velocity by working with the discrete kernel space of the divergence operator, and then prove the convergence of the pressure for the adaptive $P4/P3$ element method for the Stokes Equations. The key of the analysis is a commuting Fortin operator and a modified a posteriori error estimate. However, the construction of the Fortin operator is very complicated and the efficiency of the modified a posteriori error estimate is lack either.

During my stay in Berlin from September 10–September 18, I and my host Professor Carsten Carstensen studied convergence and optimality analysis of the adaptive $P4/P3$ element method for the Stokes Equations. The main progress is twofold. First, we proposed to use a quasi-interpolation operator for the fourth order problem to avoid the complicated Fortin operator in the aforementioned preprint. Second, by employing a Helmholtz decomposition, we showed an efficiency of the modified a posteriori error estimate.

2.2. **A New Nonconforming Finite Element Methods for the Stokes Equations in 3D.** Finite element methods (FEMs) for the two dimensional Stokes problem have been extensively studied in the literature. However only little attention has been paid to the three dimensional problem. Here, we only mention the work of D. Boffi in 1997] by the three dimensional Taylor-Hood elements. The non-conforming FEM of Crouzeix and Raviart is only suitable for the pure Dirichlet boundary condition due to a missing Korn inequality in two as well as in three dimensions. In 2D, the non-conforming FEM of Kouhia and Stenberg circumvents this by choosing only one component non-conforming and the other one conforming.

During my second stay at Mathematisches Forschungsinstitut Oberwolfach from September 19–September 24, I and Mira Schedensack proposed a new three dimensional nonconforming scheme by using the conforming linear element to approximate the first component of the velocity, the nonconforming linear element to approximate the second component of the velocity, and the conforming quadratic element to approximate the rest component of the velocity, and the piecewise constant to approximate the pressure. We proved the stability and optimality of this scheme.

2.3. **A canonical construction of H^m-nonconforming triangular finite elements.** Before I visited Germany, I and Shangyou Zhang designed a family of 2D H^m-nonconforming finite element using the full P_{2m-3} degree polynomial space, for solving $2m$th elliptic partial differential equations We have already proved the optimal order of convergence and numerically tested these new elements for solving tri-harmonic, 4-harmonic, 5-harmonic and 6-harmonic equations.

During my stay at Mathematisches Forschungsinstitut Oberwolfach, based on a draft, I finished writing the paper.

3. Others

The 5th Sino-German Workshop on Computational and Applied Mathematics was held at the University of Augsburg, Germany, from September 21st to 25th, 2015. The symposium enhanced the mutual understanding of the state of the art of current research on both sides and stimulated future Sino-German collaborations. The bilateral workshop followed previous ones in Berlin (2005), Hangzhou (2007), Heidelberg (2009), and Guangzhou (2011) chaired by Carsten Carstensen (from Humboldt-University zu Berlin) and Zhong-Ci Shi (Chinese Academy of Science, Beijing). The Augsburg workshop brought 13 Chinese scientists, 22 participants from German universities and one from a Swiss institution. One special issue was organized by soliciting 14 papers from these Chinese and German scientists in the journal Computational Methods in Applied Mathematics . During my stay in Germany, as the corresponding author, together with the other five coauthors, I wrote a short article as an epilog of the 14 preceding papers.

Finally, but not the end, motivated by my visiting and successful collaboration with my host Professor Carsten Carstensen, my current Ph.D. student Rui Ma a talent women Mathematician from the Peking University is going to apply for the postdocs level Alexander Humoldt Fellowship which will initiate a new round collaboration with Professor Carsten Carstensen and his group.

Acknowledgement

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Simons Visiting Professor Report
John Sylvester

I visited Roland Griesmaier at Universität Würzburg during the week before the Oberwolfach workshop 1638B, *Theory and Numerics of Inverse Scattering Problems.* I arrived in Würzburg on Friday, September 9th, and stayed until we left for Oberwolfach on September 18th.

During my stay, Roland and I worked daily on several projects:

We made progress on proving uncertainty estimates for the 3 dimensional far field translation operator. In a paper we finished a few weeks ago, we showed that, if an operator maps L^2 to itself, and its inverse maps L^1 to L^∞, then we can formulate an uncertainty principle related to that operator which has corollaries for several inverse problems. The dependence of the constants in those estimates on size, distance, and wavelength allows us to correctly evaluate the conditioning of the inverse problem. In two dimensions, we have theorems that describe the dependence, as well as examples that demonstrate the dependence on physical parameters is best possible. The analogous example in 3 dimensions suggested that our current estimate might not be sharp. We are now confident that the previous estimate was not sharp and see exactly what we need to do to improve it (although the proof of the new estimate may prove difficult).

Roland explained recent work with his student, where he developed a MUSIC algorithm which uses bandwidth and a single incident wave to locate an unknown number of multiple small scatterers. He outlined a program to extend this approach to image extended scatterers. We found evidence that his ongoing project already succeeds in a simple case, and expect that it will succeed more generally.

Subsequently, we formulated a program to use bandwidth in a novel way to attack the inverse source problem. One advantage of our single frequency approach is that it does data completion and source splitting with concrete sharp estimates on the conditioning of the data completion and splitting operators. It can do both data completion and far field splitting simultaneously, and we can prove that, for sources that are well-separated, the combined splitting and data completion succeeds in situations where attempting to do them separately cannot. Straightforward attempts suggest that using additional bandwidth in a naive way doesn't improve the conditioning, but physical intuition strongly suggests that the conditioning should improve if we can find a way to use it correctly. We now believe that use of prolate spheroidal wavefunction expansions in an appropriate way may well be the way to use multiple frequency information, and will begin investigating this in earnest as soon as we complete the work on 3 dimensional far field translation.

On Tuesday, we drove to Mainz to visit Martin Hanke. We explained our work on uncertainty principles to him, and he showed us a preliminary chapter in a book he is writing. The chapter deals with the Gerchberg-Papoulis algorithm, which is related to data completion and the prolate spheroidal wavefunctions in one dimension. Martin also showed us some notes extending certain formulas to two dimensions. We have since discussed these formulas, and will attempt to use them to improve estimates on the singular values of the source to far field operator which is central to our work.

This research stay was partially supported by the Simons Foundation and by Mathematisches Forschungsinstitut Oberwolfach.

Simons Visiting Professorship - Scientific Activity Report

Natalia Garcia-Fritz, University of Toronto

As a Simons Visiting Professor, I visited Universitat Autònoma de Barcelona from October 15 to October 22, 2016. My host was Professor Xavier Xarles.

This visit was related to the Oberwolfach Workshop *Definability and Decidability Problems in Number Theory* (ref. number 1643) which I attended from October 23 to October 29, 2016.

Already since the late '90, the interest on Hilbert's tenth problem (for short, H10) has gained new attention, leading to a series of H10 meetings, the most recent being the current Oberwolfach Workshop. This workshop was a unique opportunity to bring together a large group of experts working in Logic and others working in Number Theory. I learned about the new advances in the area, and also about new techniques in Number Theory that can be useful to solve decidability problems. The problem session was particularly interesting, I plan to try to work in some of the questions proposed there.

At this workshop I gave the talk *Curves of low genus on surfaces and some extensions of Büchi's Problem,* where I explained an algebraic geometric method originating in a work of P. Vojta [2] on a previous H10 meeting in 1999, further developed in my thesis [1], and I presented some new applications that I have recently obtained to problems proposed by J. Browkin and J. Brzezinski at another H10 meeting in 2010.

The aforementioned method fits into the general framework of effective approaches to Diophantine geometry, in this case function field arithmetic, which is to a large extent connected to number field arithmetic through deep conjectures of Bombieri and Lang. For the problem of proving finiteness of the set of curves of bounded genus on certain surfaces, there are useful results that are often applicable, such as Bogomolov's theorem [1]; however, these finiteness results do not shed light on the explicit determination of the set of curves of bounded genus. The latter is a serious problem because for certain arithmetic applications (e.g. Büchi's problem) finiteness is not enough and it is crucial that the low genus curves be of a precise form. The method of Vojta explained in my Oberwolfach talk points in the direction of effectiveness and it is in many cases applicable.

During my visit at Universitat Autònoma de Barcelona, I started a research collaboration with Professor X. Xarles: Our plan is to study new applications of Vojta's method for finding all curves of low genus on some surfaces. We will focus in the following case: Given two quadrics $F, G \subset \mathbb{C}[x, y, z]$ define the surface

$$S = \{sF + tG : [s : t] \in \mathbb{P}^1\} \subseteq \mathbb{P}^1 \times \mathbb{P}^2.$$

Our goal is to study relations between the geometric genus and degree of curves on these surfaces, and hopefully to find all curves of genus 0 or 1 on these surfaces, under some conditions on F and G. As of today all but one application of this method has been to study curves of low genus on smooth surfaces. This would give new applications of Vojta's method to singular surfaces.

In addition to this, during my visit I gave two talks at different seminars in Barcelona:

- *Variaciones del Segundo Teorema Fundamental de Nevanlinna-Cartan para cuerpos de funciones* (Variations of Nevanlinna-Cartan's Second Main Theorem for function fields) on October 19 at Universitat Autònoma de Barcelona.
- *Curves of low genus and k-th power consecutive values of quadratic polynomials* on October 21 at the Seminari Geometria Algebraica, Universitat de Barcelona/Universitat Politècnica de Catalunya/Universitat Autònoma de Barcelona.

This research stay was partially supported by the Simons Foundation and the Mathematisches Forschungsinstitut Oberwolfach.

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SVP report for MFO workshop "Heat Kernels, Stochastic Processes and Functional Inequalities"

M. T. Barlow

In the week 20 Nov. 2016 - 27 Nov. 2016 I visited Prof. J-D. Deuschel and his group at TU Berlin. On 23rd November I gave a probability seminar at TU Berlin, with title: "Stability of elliptic Harnack Inequality". In this seminar I described recent work with Mathav Murugan (a postdoc at UBC) which prove that the property of satisfying the elliptic Harnack inequality is stable.

During my 2 weeks in Germany I engaged in research discussions on the following topics, with researchers at TU Berlin, and then at the MFO workshop.

1. Graphs with time varying structure, and long range heat kernel bounds.

The heat kernel on a graph gives the transition probabilities for a random walk on it. Making estimates (from above and below) on the heat kernel involves a number of different regimes, in terms of the relation between time and distance. For a fixed graph, 'long range' upper bounds, i.e. which estimate the probability of moving a long distance in a short time, do not require much structural knowledge of the graph, but are an essential step in obtaining global bounds. The standard long range bounds are those obtained by Carne and Varopoulos, and later in a more general context by Davies. In the case of possibly unbounded conductances, Davies identified a metric on the graph (different from the usual graph metric) in which these bounds hold.

With Massimo Secci, Dr. Martim Slowik, and Prof. J-D. Deuschel, I discussed the question of long range bounds for time varying graphs where the edge 'conductances' $w(t, x)$ may be large. The key point is to find a good 'distance function' on the graph – as was done by Davies in the fixed graph case.

Mr Secci presented his current results. I pointed out that implicit in his calculations was an auxiliary equation on the graph, of the form

$$\frac{\partial \psi(t, x)}{\partial t} - w(t, x)|\nabla \psi(t, x)|^2 \leq 1. \tag{1}$$

Any function ψ which satisfies (1) leads to a viable space time distance function; the challenge is to find a sufficiently large solution. Our preliminary calculations suggest that the function obtained by taking equality in (1) is unlikely to be the best choice. We discussed this equation with Professor P. Friz (at TU Berlin); also at the MFO workshop the following week the connection was made with Hamilton-Jacobi equations, in a talk of E. Kopfer.

2. *Elliptic Harnack inequality (EHI) and Liouville Brownian motion*

At the MFO workshop Christophe Garban discussed the possibility of finding metrics on the sphere associated with $e^{h\beta}$, where h is the Gaussian free field. These metrics, if they exist, would play a important role in the area of conformal invariance, field theories and their connection with SLE processes. If d_h is a metric arising from the GFF in this way, and (as is expected) the associated process on the sphere, called *Liouville Brownian motion* (LBM) does satisfy heat kernel estimates with respect to this metric, the it would also satisfy the EHI with respect to this metric, as well as with respect to the standard metric on the sphere. This connection would place substantial constraints on the new metric d_h.

The EHI gives control of harmonic functions; in the abstract theory of metric measure spaces with Dirichlet form the EHI is determined by the quadratic form and the metric. It is easy to see that if d_1, d_2 are equivalent metrics (i.e. $C^{-1}d_1(x,y) \le d_2(x,y) \le Cd_1(x,y)$) then the EHI with respect to d_1 will imply the EHI with respect to d_2.

In discussions with Mathav Murugan (UBC), N. Kajino (Kobe U.), Jun Kigami (Kyoto U.) we discussed the simpler but still interesting example of the Sierpinski gasket, which is known to satisfy the EHI with respect to two distinct (non-equivalent) metrics. Kigami presented his results on the structure of these spaces; if the two metrics are quasi-symmetric this is still enough to ensure that the EHI is preserved. We asked if this condition is necessary, and will pursue this question.

In addition, I talked with Sebastian Andres about heat kernel estimates for LBM; he described his work on Brownian motion time-changed by a random field with some properties similar to the Gaussian free field, but a simpler overall structure.

3. *Elliptic Harnack inequality and p-Laplacian*

At the MFO workshop my postdoc Mathav Murugan presented our recent work on the stability of the EHI. Laurent Saloff-Coste asked if this stability also holds for harmonic functions associated with the p-Laplacian. He described the motivation for this work, and also outlined our rather incomplete knowledge at present. A fundamental starting point for the study of the EHI is the easier parabolic Harnack inequality (PHI), which was characterised by Grigoryan and Saloff-Coste in 1992, in terms of volume doubling and a family of Poincaré inequalities. For $p > 2$ these conditions are known to be sufficient for the p-PHI; necessity is still open.

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach

Dates.
Visit to TU Berlin: 20 Nov. 2016 – 27 Nov. 2016.
Attended MFO 'Heat kernels Stochastic Processes and Functional Inequalities' workshop (id. 1648): 27 Nov. 2016 – 2 Dec. 2016.

Simons Visiting Professorship Scientific Activity Report

by Zhen-Qing Chen

I have visited Professor Jean Dominique Deuschel at TU Berlin as a Simons Visiting Professor from Nov. 20, 2016 to Nov. 27, 2016, before moving to Oberwolfach for the MFO workshop "Heat Kernels, Stochastic Processes and Functional Inequalities" held from November 28 to December 3, 2016.

During my visit at TU Berlin, I have discussed mathematics of common interest with Professor Deuschel and his group including Dr. Martin Slowik and his Ph.D students. More specifically, we have discussed random walk in random environments, heat kernel estimates, Moser's and DeGiorgi's iteration methods in obtaining a priory Hölder's estimates for harmonic functions. With Jean Dominique, Tuan Nguyen, Martin Slowik and Martin Barlow, I have also discussed some inequalities of random walk in square lattices from the point of view from the Dirichlet-to-Neumann map in domains. The latter formulation allows one to show that for every $p > 1$, $(\int_{\mathbb{R}^{n-1}} |\frac{\partial}{\partial x_n} H\phi(x,0)|^p dx)^{1/p}$ is comparable to $(\int_{\mathbb{R}^{n-1}} |\phi(x)|^p dx)^{1/p}$ for every smooth function ϕ on $\mathbb{R}^{n-1}$, where $H\phi(x)$ is the harmonic extension of ϕ on $\mathbb{R}^{n-1}$ to the upper half space $\mathbb{R}^{n-1} \times [0, \infty)$. This is because the normal derivative $\frac{\partial}{\partial x_n} H\phi(x,0)$ on the boundary of the upper half space in $\mathbb{R}^n$ can be identified with the fractional Laplacian $-(-\Delta)^{1/2}\phi(x)$ on $\mathbb{R}^{n-1}$.

Martin Barlow from UBC was also visiting TU Berlin at the same time as a Simons Visiting Professor. He and I have discussed on Harnack inequalities and their stability. Both of us have recently been working on this subject but for different type of processes. He and Mathav Murugan studied elliptic Harnack inequalities for symmetric diffusion processes, while in joint work with Takashi Kumagai and Jian Wang, I have been studying parabolic and elliptic Harnack inequalities for symmetric jump processes.

In the afternoon of Wednesday November 23, 2016, I gave a RTG Seminar/Berliner Colloquium at TU Berlin titled "Harnack inequalities for symmetric non-local Dirichlet forms and their stability". Mathematicians from TU Berlin and from Humboldt University attended this joint probability seminar. The talk is based on recent joint works with T. Kumagai and J. Wang. In the talk, I mainly focused on the stability of parabolic Harnack inequalities for symmetric pure jump processes. Consider two symmetric Hunt processes X and $\widetilde{X}$ of pure jump type associated with two regular Dirichlet forms on a locally compact metric measure space M. The process X is symmetric with respect to a σ-finite Radon measure μ on M with jumping intensity kernel $J(x,y)$ with respect to $\mu \times \mu$, while the process X is symmetric with respect to a σ-finite Radon measure $\widetilde{\mu}$ on M with jumping intensity kernel $\widetilde{J}(x,y)$ with respect to $\widetilde{\mu} \times \widetilde{\mu}$. Suppose μ satisfies volume doubling and reversed volume doubling property, and there are constants $c_2 > c_1 > 0$ so that

$$c_1\widetilde{\mu} \leq \mu \leq c_2\widetilde{\mu} \quad \text{and} \quad c_1\widetilde{J}(x,y) \leq J(x,y) \leq c_2\widetilde{J}(x,y).$$

Let ϕ be a continuous increasing function with $\phi(0) = 0$ and $\phi(1)$ satisfying the doubling and reversed doubling property. We show that parabolic Harnack inequality with scaling function ϕ (denoted as PHI (ϕ)) holds for X if and only if it holds for $\widetilde{X}$. This stability result result is a direct consequence of a more precise characterization of PHI(ϕ) we got recently in terms of a Sobolev inequality, a cutoff energy inequality, an upper bound and an averaging property for the jumping kernel $J(x, y)$. Other equivalent characterizations for PHI(ϕ) in terms of heat kernel estimates, mean exit time bounds, Hölder regularity of parabolic functions and harmonic functions, as well as its relation to elliptic Harnack inequalites are also given in the talk.

On November 24, 2016, I also attended a weekly seminar of the Research Unit on Rough paths, stochastic partial differential equations and related topics at TU Berlin. The seminar was given by Milton Jara from IMPA on "Non-equilibrium fluctuations of interacting particle systems".

On Sunday November 27, 2016, I moved from Berlin to Oberwolfach to attend MFO Workshop ID 1648 on "Heat Kernels, Stochastic Processes and Functional Inequalities". It was a stimulating conference, where I met many old friends and got acquainted with new ones. I gave a 40 minutes talk on Stability of parabolic Harnack inequalities for symmetric non-local Dirichlet forms on Monday November 28, 2016 at the workshop. At the Oberwolfach Workshop, I continued discussions with Professor Deuschel on heat kernel estimates, random walks and Brownian motions in random environments. I also had many discussions with other participants during the workshop.

It was a fruitful two weeks stay in Germany. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

SCIENTIFIC ACTIVITY REPORT

ANURAG K. SINGH

I participated fully in the Oberwolfach Workshop *Asymptotic phenomena in local algebra and singularity theory* that took place December 11–17, 2016. Following the workshop, I visited the University of Genova (Italy) under the Simons Visiting Professorship program for the period December 18–24, 2016.

My host at the University of Genova was Aldo Conca. During this visit, I collaborated extensively with Conca and Matteo Varbaro (also at the University of Genova) on a project related to Hankel determinantal rings. Other collaborators on this project are Maral Mostafazadehfard and Kei-ichi Watanabe. A few details are outlined later in this report.

Seminar: At Genova, I presented a lecture on *Magic Squares*. A magic square, in this context, is a matrix with nonnegative integer entries such that each row and column has the same sum, the line sum. It was conjectured in [ADG] that the number of $n \times n$ magic squares with line sum r agrees with a degree $(n-1)^2$ polynomial in r for all integers $r \geqslant 0$. This was proved by Stanley [St], and was arguably the starting point of combinatorial commutative algebra. In my lecture, I gave a self-contained proof of Stanley's theorem using positive characteristic techniques. The key ideas come from tight closure theory and the F-regularity of the underlying affine semigroup ring.

The topic leads into areas of active research: following his proof regarding the enumeration of magic squares, Stanley asked if the h-vector of the affine semigroup ring associated with magic squares is unimodal. This was indeed proved to be the case by Athanasiadis [At] thirty years later, and leads to a much more general question: for standard graded Gorenstein rings with rational singularities, is the h-vector always unimodal? This was one of several topics of discussion with Conca and Varbaro at Genova.

Outreach: In addition to the seminar above, which was well-attended by students, I also presided over the annual departmental award ceremony at the University of Genova.

Hankel determinantal rings. Determinantal rings are classical objects of study in commutative algebra: they arise naturally as rings of invariants, and a study of their properties has driven a substantial amount of work in the field. Hankel determinantal rings are linear specializations of determinantal rings that arise as the higher order secant varieties of rational normal curves; they have been studied extensively by Gruson and Peskine [GP], Eisenbud [Ei], Conca [Co], and others. While their Cohen-Macaulay property follows from that of determinantal rings [HE], it is not known whether, in general, Hankel determinantal rings are invariant rings of reductive groups acting linearly on polynomial rings. Thus, results such as Boutot's theorem do not apply to Hankel determinantal rings; this makes Theorem 1 below particularly interesting.

By a result of [GP], every Hankel determinantal ring is isomorphic to one where the defining ideal is generated by minors of maximal size; in view of this, by a *Hankel matrix*

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we will mean here a $t \times n$ matrix of the form

$$H = \begin{pmatrix} x_1 & x_2 & x_3 & \cdots & x_n \\ x_2 & x_3 & \cdots & \cdots & x_{n+1} \\ x_3 & \cdots & \cdots & \cdots & x_{n+2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_t & \cdots & \cdots & \cdots & x_{n+t-1} \end{pmatrix},$$

where $x_1, \ldots, x_{n+t-1}$ are indeterminates over a field $\mathbb{F}$. By a *Hankel determinantal ring* we mean a ring

$$\mathbb{F}[x_1, \ldots, x_{n+t-1}]/I_t(H),$$

where $I_t(H)$ is the ideal generated by the size t minors of H. One of our results is:

Theorem 1. *Let R be a Hankel determinantal ring over a field of characteristic zero. Then R has rational singularities.*

Our proof uses reduction modulo p techniques, and a theorem of Smith [Sm]. While we also expect that Hankel determinantal ring of positive characteristic are F-regular, we have only been able to verify special cases of this so far, such as:

Theorem 2. *Hankel hypersurfaces of positive characteristic are F-regular.*

The general case remains a topic of our investigations. On the other hand, we prove in full generality that the divisor class group of a Hankel determinantal ring is cyclic:

Theorem 3. *For $\mathbb{F}$ a field, the Hankel determinantal ring $R = \mathbb{F}[x_1, \ldots, x_{n+t-1}]/I_t(H)$ is $\mathbb{Q}$-Gorenstein, with divisor class group isomorphic to $\mathbb{Z}/(n-t+2)$.*

Another striking find is that each element of the divisor class group of R, as above, is a maximal Cohen-Macaulay module; what makes this striking is that this is not the case for classical determinantal rings. We have also made substantial progress towards understanding all elements of the divisor class group: this now appears to be within reach.

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Simons Visiting Professorship

Cameron M Gordon

Scientific Activity Report

The host university was Johannes-Gutenberg Universität Mainz, and the host was Professor David E. Rowe. I was there from December 12–18, 2016. I was at Oberwolfach from December 18–23, participating in the Mini-Workshop "Max Dehn: his Life, Work, and Influence".

In Mainz I gave a colloquium on December 15 entitled "Left-orderability and cyclic branched covers of knots". The context here is a recent conjecture in 3-dimensional topology that relates three very different aspects of 3-manifolds: orderability properties of the fundamental group (algebraic), codimension 1 foliations (geometric), and Heegaard Floer homology (essentially analytic). More precisely, the conjecture is that for a prime 3-manifold M, the following are equivalent: (LO) $\pi_1(M)$ is left-orderable, (CTF) M supports a co-orientable taut foliation, and (NLS) M is not a Heegaard Floer L-space. (An L-space is a 3-manifold whose Heegaard Floer homology is "minimal".) I discussed this in the case where M is the n-fold cyclic branched cover $\Sigma_n(K)$ of a knot K. Interest in this case comes partly from the fact that one of the first large classes of L-spaces discovered were the 2-fold branched covers of alternating knots. Subsequently these were shown to be non-LO. The talk focused on the 2-bridge knots $K = K_{p/q}$ (here p/q is a rational number). These are alternating, and in fact $\Sigma_2(K)$ is the lens space $L(p, q)$, so it clearly fails to satisfy all three conditions LO, CTF, and NLS. However, the behavior of $\Sigma_n(K)$ for $n > 2$ varies. For example, for infinitely many p/q, $\Sigma_n(K)$ fails to satisfy the conditions for all n. On the other hand, for infinitely many p/q, $\Sigma_n(K)$ becomes LO for n sufficiently large. In investigating this I recently proved a conjecture of Riley, dating from the early 1970s, that the number of non-abelian $SL(2, \mathbb{R})$ representations of the group of a 2-bridge knot K is at least half the absolute value of the signature $\sigma(K)$ of K. So if $\sigma(K) \neq 0$ then there is at least one such representation and one can then show that $\Sigma_n(K)$ is LO for n sufficiently large. An obvious problem is to show that $\Sigma_n(K)$ is also CTF and NLS. Under the stronger assumption that $|\sigma(K)| = 2g(K)$, Michel Boileau, Steven Boyer and I have shown that $\Sigma_n(K)$ is NLS (for n sufficiently large); the CTF property remains unknown. The known results suggest that either $\Sigma_n(K)$ is non-LO for all n, or there is some n_0 such that $\Sigma_n(K)$ is non-LO for $n \leq n_0$ and LO for $n > n_0$. Proving this, and finding the cut-off point n_0 for a given p/q, is an interesting open problem.

In Mainz I also spent some time discussing with David Rowe various aspects of Max Dehn's work. In the projected book on Dehn that was the subject of the Oberwolfach Mini-Workshop, I plan to write a chapter on Dehn's 1910 paper *Über die Topologie des dreidimensionalen Raumes*. One of the main things in this paper is Dehn's famous *"Lemma"*, asserting that a loop in a 3-manifold that bounds a singular disk with no singularities on its boundary also bounds a non-singular disk. In 1929 Hellmuth Kneser wrote to Dehn pointing out that his proof was flawed, and an interesting correspondence ensued. (Proving Dehn's "Lemma" became a central problem in 3-dimensional topology, finally achieved by Papakyriakopoulos in 1957.) Much of the Dehn-Kneser correspondence is in the Dolph

Briscoe Center for American History at the University of Texas at Austin, but has never been published. I brought copies of the letters to Mainz and David Rowe began translating them. We discussed them at length. They provide fascinating and surprising insights into this important episode in the history of 3-dimensional topology, as well as into the personality of Dehn.

The organizers of the Oberwolfach Mini-Workshop were David Peifer, Volker Remmert, David Rowe, and Marjorie Senechal. The goal was to discuss plans for the book on Dehn mentioned above. Several chapters are planned, covering all aspects of his life, including the details of his mathematical career, the amazing story of his escape from Germany in 1939 to (eventually) the US, his time at Black Mountain College in North Carolina, his interest in history and botany, and so on. There were some formal talks, but a lot of the time was spent in group discussions. I gave an informal talk on Dehn's Lemma, explaining its significance for 3-dimensional topology and discussing the error in Dehn's proof. I continued working on the Dehn-Kneser correspondence, with David Rowe and Cynthia Hog-Angeloni, trying to understand the counterexamples that were being discussed in the letters. (The issue is complicated inasmuch as Papakyriakopoulos' proof shows that in fact Dehn's method does always work: the problem is that Dehn's proof that it always works was incomplete.) The Workshop was very interesting, with the participants coming from several varied backgrounds. A highlight was the presence of Trueman MacHenry, who had been a student of Dehn in mathematics (one of only two) at Black Mountain College, and who was therefore able to share many personal reminiscences. Out of the Workshop we began to get a coherent picture of the many facets of Dehn, as well as a fairly clear idea of the overall structure of the book and a timetable for its completion. Finally, in addition to the discussions about Dehn, I worked with Cynthia Hog-Angeloni on some unrelated problems on embeddings of 2-complexes in 3-manifolds, in connection with a chapter she is writing, with Janina Glock and Sergei Matveev, in a book "Advances in two-dimensional Homotopy and Combinatorial Group Theory", edited by Wolfgang Metzler and Stephan Rosebrock, which will appear in the London Mathematical Society Lecture Note Series.

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Links to the Oberwolfach Reports of the 16 hosting workshops 2016

Workshop: 1605 Algebraic Cobordism and Projective Homogeneous Varieties
http://www.mfo.de/occasion/1605/www_view

Workshop: 1607a Topological Recursion and TQFTs
http://www.mfo.de/occasion/1607a/www_view

Workshop: 1608b New Developments in Functional and Highly Multivariate Statistical
http://www.mfo.de/occasion/1608b/www_view

Workshop: 1617 Moduli spaces and Modular forms
http://www.mfo.de/occasion/1617/www_view

Workshop: 1625 Hyperbolic Techniques in Modelling, Analysis and Numerics
http://www.mfo.de/occasion/1625/www_view

Workshop: 1626 Algebraic K-theory and Motivic Cohomology
http://www.mfo.de/occasion/1626/www_view

Workshop: 1627a Statistics for Shape and Geometric Features
http://www.mfo.de/occasion/1627a/www_view

Workshop: 1629 Topologie
http://www.mfo.de/occasion/1629/www_view

Workshop: 1631 Computational Group Theory
http://www.mfo.de/occasion/1631/www_view

Workshop: 1636 Self-Adaptive Numerical Methods for Computationally Challenging Problems
http://www.mfo.de/occasion/1636/www_view

Workshop: 1638a Adaptive Algorithms
http://www.mfo.de/occasion/1638a/www_view

Workshop: 1638b Theory and Numerics of Inverse Scattering Problems
http://www.mfo.de/occasion/1638b/www_view

Workshop: 1643 Definability and Decidability Problems in Number Theory
http://www.mfo.de/occasion/1643/www_view

Workshop: 1648 Heat Kernels, Stochastic Processes and Functional Inequalities
http://www.mfo.de/occasion/1648/www_view

Workshop: 1650 Asymptotic Phenomena in Local Algebra and Singularity Theory
http://www.mfo.de/occasion/1650/www_view

Workshop: 1651b Max Dehn: his Life, Work, and Influence
http://www.mfo.de/occasion/1651b/www_view